THICK-LENS OPTICS

AN ELEMENTARY TREATISE FOR THE STUDENT AND THE AMATEUR

BY

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PREFACE

This volume is the outcome of an attempt to answer certain questions regarding the optics of the microscope and telescope; questions to which no thoroughly satisfactory answers could be found in any literature accessible to the author.

Many answers were found, but they were discordant and unusable for practical work, mainly by reason of their complexity and seeming contradictoriness and lack of co-ordination.

The following pages seek to answer these questions in a manner so plain and simple that the average amateur can find out for himself what is going on optically in his camera, microscope, or telescope.

To this end the mathematics is of the simplest kind, so that the busy man who has forgotten all or most of his mathematics can nevertheless work his way through, provided he can use the simplest kind of algebra, two theorems in elementary geometry and one in trigonometry. For the reader who has not had trigonometry, the few simple principles required are given in the text. So far as mathematical difficulties go, any high-school student is sufficiently equipped.

As an aid to concreteness and clearness the investigations are based upon graphic principles as much as possible and along intuitive lines.

For the more inquisitive reader who desires a more

rigidly logical basis one investigation is given in analytical form, as a supplement to the preceding intuitive ones. This can safely be omitted by those not interested, without destroying the continuity of the text.

The text is a working one, intended to give the reader practical and intelligible rules of procedure, with full and thorough explanations, so that the most cursory reader can utilize them. Many practical examples are fully worked out and many more given for practice.

Particular pains has been taken to reconcile seemingly contradictory formulae for the same result, which, unreconciled, leave the reader in the deepest uncertainty, the fault of most of the literature on the subject.

This volume, for the first time apparently, assembles these rules, answers, and formulae in one consistent whole, in a practical form intelligible to the non-technical reader.

The formulation of the methods of procedure is so standardized and simplified (§ 106) that it is expected that the reader can readily utilize the necessary calculations, concretely visualized and checked by the graphic constructions (§ 107).

His ability to do so ought to render his use of optical instruments that much more intelligent and interesting, and enable him to know roughly how his instrument is doing its work, what effect a change of lens or of its position would have, how to decide in a rough way what form of lens he wants for certain effects, and how he could modify those effects.

The investigation is for a single monochromatic ray, and therefore the questions of achromatic and spherical aberration are not touched upon, as not being within the scope of the simple treatment used.

The goal of the work is, of course, the practical calcula-

tions of §§ 106, 107 and Chapter V, though many other important calculations are gathered on the way.

To render the work less an isolated monograph and make it more useful to the general reader, a number of sections have been added, to round out the subject somewhat toward the nature of a handbook and to increase its practical value to the owner of an optical instrument; not the least valuable of which will be the chapter on Experimental Observations. This chapter will enable the reader to get an experimental acquaintance with the optical constants of his lenses.

The author makes little claim to novelty, except in the simplification and workability of the rules of procedure.

A. L. B.

Brooklyn, N. Y.
October 1, 1912.

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THICK-LENS OPTICS

CHAPTER I

SURFACE REFRACTION

1. As this is intended as a working manual, and not a treatise, it will be assumed at the outset that the reader is acquainted with the fundamental principles of optical refraction, the commonplaces of the elementary text-books, viz.:

(*a*) Light rays are propagated in straight lines.

(*b*) In passing from one medium to another, a ray of light is deflected towards the normal to the surface in passing into the denser medium; vice versa in passing out.

2. Definition. — The ratio of rise to slant of a line is called the *sine* of the angle of inclination; of rise to run, the *tangent* of the angle of inclination.

For example: In a roof whose vertical height (rise) is 3 feet, whose half width (run) is 4, the length of the rafters (slant) will be 5, and $\frac{3}{5}$ is the *sine* of the angle of inclination of the roof to the horizon; $\frac{3}{4}$ is the *tangent* (generally written tan) of the angle of inclination, ratio of rise to run.

3. Definition. — The ratio of the sine of the angle of incidence to the sine of the angle of refraction is called the *index of refraction* for the denser medium, the ray passing into the denser medium from air.

4. Surface Refraction.

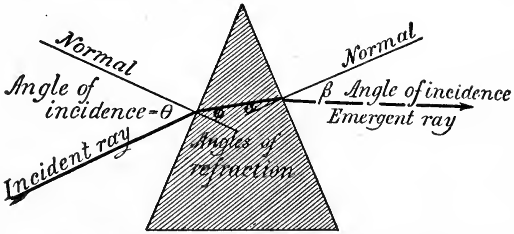


Diagram showing angles of incidence, refraction, etc.

The index of refraction is generally indicated by the Greek letter μ, thus:

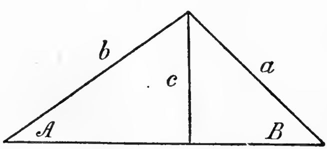
$$\mu = \frac{\sin \theta}{\sin \phi} \qquad \mu' = \frac{\sin \beta}{\sin \alpha}$$

Unless otherwise specified, we will consider only two media: air and glass.

5. Law of Refraction of Light. — The index of refraction for the same two media is constant, whatever the angle of incidence.

6. Trigonometric Law of Sines. — In any triangle, the sides are proportional to the sines of the opposite angles.

Proof. By definition of sine,



$$\sin A = \frac{c}{b}$$

$$\sin B = \frac{c}{a}$$

Whence, by division,

$$\frac{\sin A}{\sin B} = \frac{a}{b} \qquad \text{Q.E.D.}$$

7. Note. — In all construction diagrams, the order of construction is indicated by the alphabetical order of the letters; e.g. in the diagram below, draw the circle a with the radius indicated, then draw the circle b, then locate the point C, then locate the point D, and then, since this ends the series of letters, draw the refracted ray as shown.

8. Construction for Surface Refraction.

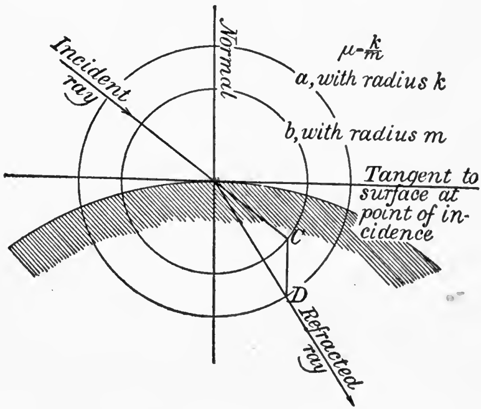


Diagram showing construction for surface refraction. Given the incident ray, to find the refracted ray.

For refraction out, interchange the letters C and D.

9. Definition. — Any arbitrary line through the center of curvature is called the *axis* of the surface. The point where it pierces the surface is called the *vertex*.

10. Convention as to Signs. — Distances measured to the right from the vertex are considered as positive; those to the left negative.

Note. — In the following investigations, the diagrams will usually be so taken as to make all the elements con-

sidered positive. This will give *normal equations which may be considered typical for all cases.*

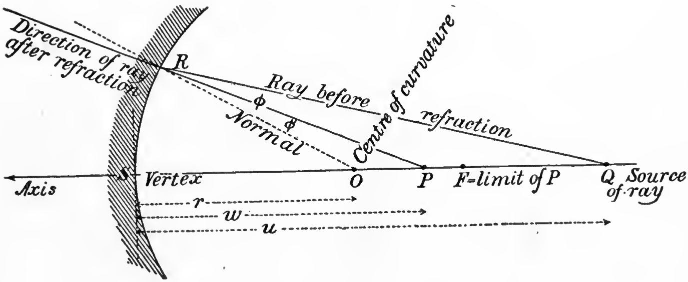
11. Equation for Surface Refraction (incident onto denser medium).

By the Law of Sines (§ 6),

$$\frac{QO}{QR} \sin QOR = \sin \phi$$

$$= \mu \sin \phi'$$

$$\left[\mu = \frac{\sin \phi}{\sin \phi'} = \frac{\text{sine of incident angle}}{\text{sine of refracted angle}}\right.$$



$$\frac{PO}{PR} \sin POR = \sin \phi'$$

$$\therefore \quad \frac{QO}{QR} \sin QOR = \mu \frac{PO}{PR} \sin POR$$

$$\therefore \quad QO \cdot PR = \mu \cdot PO \cdot QR$$

[$\sin QOR = \sin POR$

As R approaches S, P approaches some point F as a limit.

$$\therefore \quad (u - r)\, w = \mu\, (w - r)\, u$$

or, in the more usual form,

$$\frac{\mu}{w} - \frac{1}{u} = \frac{\mu - 1}{r}$$

$$\left[\begin{array}{l} u = \text{dist. from vertex to source, + to right} \\ w = \text{dist. to image, + to right, − to left} \\ r = \text{radius of curvature, + to right} \\ \mu = \text{index of refraction}^{1} \end{array}\right.$$

12. When the source is very distant, i.e. $u = \infty$, w takes the special value f, and

f = distance to point through which rays parallel to the axis in the rarer medium meet the axis after refraction

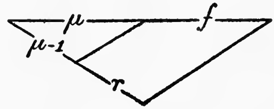
= focal radius of the surface for parallel entering rays

$= \dfrac{\mu r}{\mu - 1}$ (r meas. from vertex to right is pos. and vice versa).

N. B. — This formula holds whether light comes from right or left, or surface convex or concave.

13. For emergent rays $f_o = \dfrac{-r}{\mu - 1}$ = focus for rays parallel in the denser medium. Note how it differs from that of § 12.

14. Graphic Check. — Check the calculation of f by similar triangles drawn to scale, in which the sides are represented as shown. This will readily detect large errors of calculation in time to prevent their vitiating later calculations.

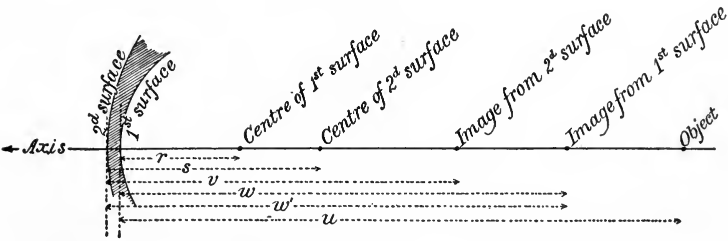


[1] The μ represents the ratio of the sine of the angle of incidence to the sine of the angle of refraction. In the case of incidence on a denser medium it will be the index of refraction and an improper fraction, but in the case of incidence on a rarer medium it will be the reciprocal of the index of refraction and a proper fraction.

CHAPTER II

THIN LENSES

15. By thin lenses is meant lenses whose thickness can be practically neglected in comparison with the other elements under consideration.



By § 11, for the first surface,

$$\frac{\mu}{w} - \frac{1}{u} = \frac{\mu - 1}{r} = \frac{\mu}{f}$$

Similarly, for the second surface, the image of the first surface being the object of the second surface, and the index for the second surface being inverted, because the ray is emergent instead of incident (see § 11, footnote),

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{w'} = \frac{\frac{1}{\mu} - 1}{s}$$

[For significance of the letters see the diagram

or $$\frac{\mu}{w'} - \frac{1}{v} = \frac{\mu - 1}{s} = -\frac{\mu}{f'}$$

The negative sign is used merely to make the formula below conform in looks to that of § 91 and similar ones. This convention has no effect either on the numerical value of F (§ 16) or on the sign of F.

If the lens is thin, so that practically $w = w'$, then, by subtraction,

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

$$= \mu\left(\frac{1}{f} + \frac{1}{f'}\right)$$

r = radius of first surface
s = radius of second surface
f = focal length of first surface for incident rays
f' = negative focal radius of the second surface (see preceding paragraph) $= \dfrac{-\mu s}{\mu - 1}$
μ = index of refraction

16. For a very distant source, i.e. $u = \infty$, we get the special value for $\frac{1}{v}$.

$$\frac{1}{F} = \mu\left(\frac{1}{f} + \frac{1}{f'}\right) = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$F = \frac{ff'}{\mu(f + f')}$$

F = principal focal length of the lens
= value of v for $(u = \infty)$

That is, for a very distant object (horizontal rays) all the horizontal rays pass through F, the focal point.

17. Since distances from the vertex to the left are negative, we get

For a double convex lens

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(-\frac{1}{\mathbf{r}} - \frac{1}{\mathbf{s}}\right)^{1} = \frac{1}{F} = \frac{1}{\mathbf{v}} + \frac{1}{\mathbf{u}}$$

[1] Heavy black face type indicates numerical values without regard to direction; light letters indicate true values, taking account of direction where this is necessary. The black face type will be used when

For a double concave lens

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{\mathrm{r}} + \frac{1}{\mathrm{s}}\right) = \frac{1}{F} = \frac{1}{\mathbf{v}} - \frac{1}{\mathbf{u}}$$

18. Starting with $u +$ and large, if F is $+$, $\frac{1}{v}$ must be greater than $\frac{1}{u}$, in order to make $\frac{1}{v} - \frac{1}{u}$ positive. Therefore v must be smaller than u, and the image is nearer the lens than the object is.

If $r > s$, thus making the lens thicker in the middle, or if $r < 0$, i.e. negative, thus making the lens a double convex lens, then $\frac{1}{F}$ is negative, and therefore F is negative and must lie on the left.

Keeping $u +$ (or on the right) and large, which makes $\frac{1}{u}$ small, the only way to make $\frac{1}{v} - \frac{1}{u} < 0$ (i.e. neg.) is to make v negative.

In other words, for a thinner-in-the-middle lens, later called a negative lens, u, F, and v have the same sign when the object is real.

19. For a thicker-in-the-middle lens, later called a positive lens, and a real object, F and v must have different signs from u.

Stated in another way,

For a $+$ lens, the further focus (from the object) is the active focus.

For a $-$ lens, the nearer focus is the active focus.

20. Light from the Right. — In formulae used hereafter, the positive lens (thicker in the middle) will be considered

the absence of direction is to be specially emphasized. In other cases the context will indicate whether the quantities have direction or not.

as having a negative focal length, and negative lenses (thinner in the middle) as having a positive focal length, because the effective focus lies in these respective directions.

21. Light from the Left. — In future formulae, the positive lens will be considered as having a + focal length, and the negative lens as having a − focal length, because the effective focus lies in these directions respectively.

Since this makes the sign of the lens and the sign of the focal length concordant (a great gain in uniformity), *the light will hereafter be assumed to come from the left* unless otherwise specified.

Note. — The diagram in § 15 was so taken because all the quantities are positive, thus giving a normal formula (see § 10, note) applicable to any diagram when we take account of the changes in sign of the various quantities. So also in diagrams of §§ 67, 72.

22. Notice that if the media on the two sides of the lens are not the same, the nearer and farther focal-point distances will be different (Conf. § 71). This will easily be seen by, in the preceding investigation, taking μ' for the second surface instead of μ, and finding F for $u = \infty$. Reversing this by taking μ' for the first surface and μ for the second surface, we get a different value for F.

Unless specifically mentioned, we assume that we have air on both sides of the lens, the usual condition, and therefore the two focal-point distances the same.

23. Optical Center. — The investigation of § 65, which may be read here, shows that for any lens there is a point, rays passing through which are parallel before and after refraction by the lens. For a thin lens this point must be where the axis of the lens pierces the lens. Rays through this point are not changed in direction.

24. Diagrammatic Investigations. — In the diagram-

matic investigations which follow, the determination of image from object will be made by means of two definite kinds of rays.

(*a*) One which after refraction passes through the proper focus point, i.e. rays from the object parallel to the axis of the lens, as if from a distant object.

(*b*) One unchanged in direction before and after refraction, i.e. a ray through the center of the lens (or in the case of a thick lens, the nodal points, see § 63).

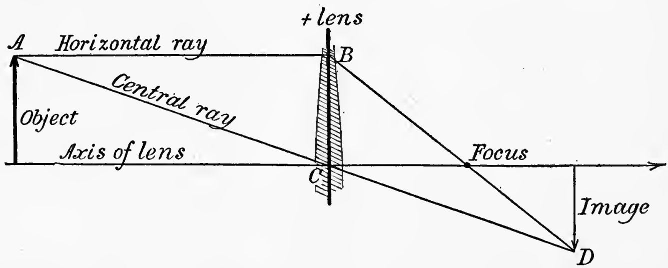
To emphasize the characteristics, say,

Horizontal rays always refract to the focus.

Central (or in the case of a thick lens, nodal, see § 63) rays pass through without angular deviation.

Note. — Each point in the object sends rays in all directions, and of course we choose those which serve us best.

25. Diagrammatic Derivation of Image (object outside of focal distance).



Note the order of the letters (see figure); this order, formulated, becomes a rule of procedure.

Convention. — The object is represented by a heavy arrow, the image by a light arrow, the focus by F, the thin lens by a vertical heavy straight line.

AB represents one of the first kind of rays, which, we know, must go through F; AC represents the second

kind, which goes through without refraction. Hence D, the point where the two rays meet, must correspond to A, or be the image of A.

The diagrammatic procedure of finding the image D, of an object A, may be symbolized by the letters (a great help in future diagrams)

$$hl \rightarrow f_2 \text{ to } c$$

meaning:

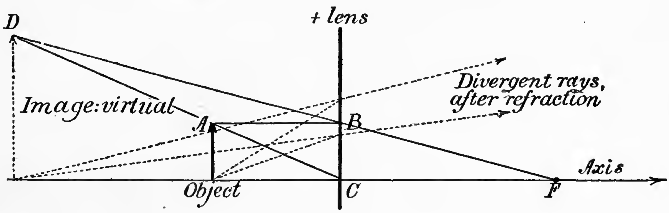
From some point in the object pass horizontally (h) to the lens (l), then through ($\rightarrow$) the right-hand focus (f_2) to the center line (c). The intersection will be the image point. (f_1 would mean left-hand focus.)

Observe that F and v are $+$, while u is $-$. Compare with § 19.

26. In a similar way we get the following diagrams.

When **u** > **F** numerically, we get a real aerial image which can be made visible by interposing at the image point a piece of ground glass. The rays from the object to the lens are *divergent rays*, those from the lens to the image are convergent rays.

27. Virtual Image (object inside the focal distance).

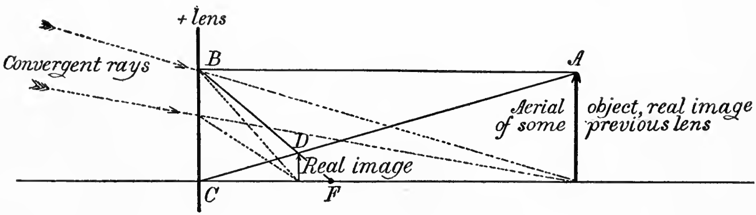


Object inside the focal distance: the dotted lines show rays made less divergent after refraction.

Diagram showing how an object within the focal point gives a virtual image, an image erect instead of inverted as in the previous diagram, and which cannot be made visible by the interposition of a piece of ground glass. Unlike the previous case, it renders the divergent rays less divergent, but not convergent.

28. Convergent Rays.

Notice that the converging rays are rendered more converging.

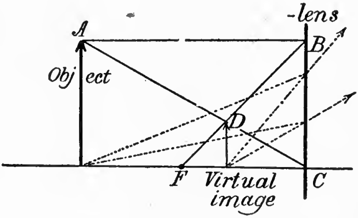


Aerial object outside or inside the focal distance.

Diagram showing the effect of interposing a + lens in the path of converging rays, thus producing a real image.

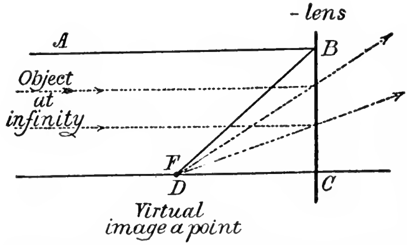
N. B. — In tracing images, notice what a different result we get for the same position of the object, influenced by its being a real, or an aerial object with converging rays. This is of great importance in tracing images.

29. Negative Lens. — In the same manner we get the

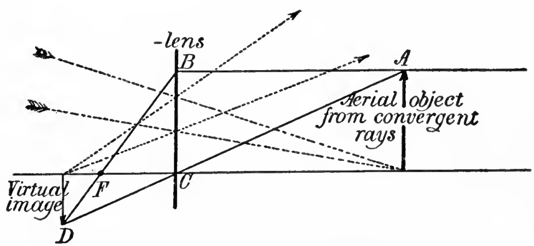


Divergent rays: virtual image.

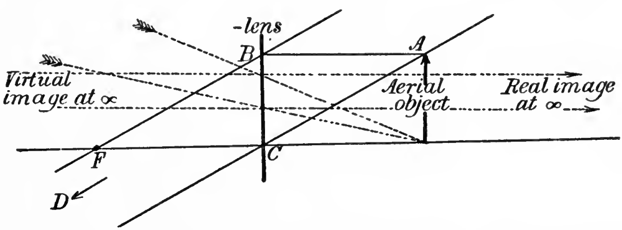
following progressive diagrams, showing the result of moving the object outward at the left until it disappears at ∞, coming in again at the right from ∞ and moving down to within the focal distance.



Parallel rays: virtual image a point



Convergent rays: aerial object outside the focal distance: virtual image.



Convergent rays: aerial object at focal distance: image virtual or real, at infinity.

Note. — The letters $ABCD$ have the same progressive signification as in § 25.

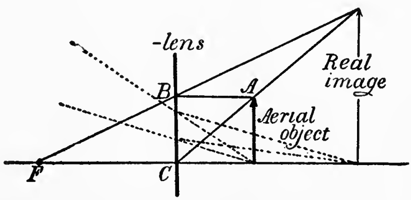
Notice that the formula has now become $hl \rightarrow f_1$ to c.

30. Collecting these results, we have the following formulation of procedure.

Formula for diagram tracing of rays from object to image:

Positive lens	$hl \rightarrow f_2$ to c
Negative lens	$hl \rightarrow f_1$ to c

For explanation of symbols, see § 25.



Convergent rays: aerial object inside focal distance: real image.

Notice, what has been elsewhere spoken of, the effective focus for a + lens is the right-hand one; for a − lens, the left-hand one. (Light from the left.)

31. In some cases it becomes necessary to trace back the rays from the image to the object (see § 37, Ex. 13), in which case we have:

Formulation of procedure for diagram tracing of rays backward from image to object:

Positive lens	$rf_2l \parallel cf_2$ to c
Negative lens	$rf_1l \parallel cf_1$ to c

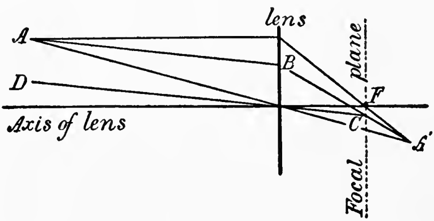
meaning, draw a ray (r) through f_2 to the lens (l), and then along a parallel to a center line through f_2 ($\parallel cf_2$) to the center line (c). f_1 is the left-hand, f_2 the right-hand focus.

The use of this formulation will be found to be of great help in tracing graphically the conjugate points of a lens. In fact, without the mechanical aid of the formulation,

it is exceedingly difficult at times for the novice to do so without error, especially in tracing from image to object.

32. Diagram for Oblique Rays.

By § 25 A' is the image of A. A second ray from A, AB, must go to the same point, A'. But 34 by § ~~33~~ it must also go through the point C, where the parallel center line DC pierces the focal plane.



But we can consider the ray AB as an oblique ray, and the formula for oblique rays is evidently

$$rl \rightarrow \phi_2 \parallel \text{to } c$$

meaning, draw a ray (r) to the lens (l), and then through ($\rightarrow$), the secondary focus determined by a parallel through the center intersecting with the focal perpendicular ($\phi_2 \parallel$), to the center line from the object (c). Omission of "to c" gives the direction of the refracted ray, independent of the origin.

This formula evidently includes that of § 25 as a particular case.

The reverse formula for tracing from image to object is $r\phi_2 l \parallel c\phi_2$ to c, the interpretation of which is similar to that of § 31, of which this is the general case.

For a *negative* lens, the corresponding formulae are

$$rl \rightarrow \phi_1 \parallel \text{to } c \quad \text{and} \quad r\phi_1\, l \parallel c\phi_1 \text{ to } c$$

This is one of the most important sections in the book for enabling the investigator to get a quick and graphic idea of the location of the images due to a succession of lenses,

allowing, as it does, any ray to be traced, whatever the effect of the lens upon it.

Start the ray from the intersection of the object with the axis. Each new intersection with the axis will locate an image.

The same principle applies to the refraction through a surface and reflection from a surface, the surfaces being typified by vertical straight lines, as in the case of lens surfaces.

Since the point A' lies on the line AA' through the optical center, its position will not be changed by twisting the lens about a vertical axis through the optical center. This will have an important bearing in subsequent sections.

The reverse formulation (see § 31) is $r\phi_2 l \parallel c\phi_2$ to c, and $r\phi_1 l \parallel c\phi_1$ to c, for + and − lens respectively.

33. As an example of the application of the section

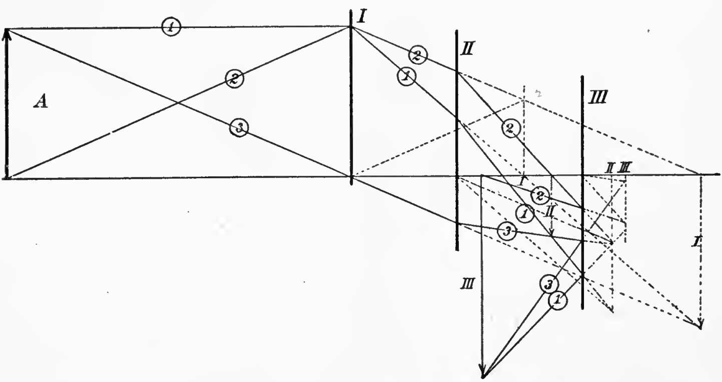


Diagram illustrating the tracing of oblique rays from the object A, through the three lenses, I, II, III, to the final image, III. The foci and images of the lenses are marked correspondingly I, II, III. The rays in their different courses are marked by encircled numerals. This is, in a very distorted form, the course through a compound microscope, I being the objective and II, III the lenses of the eyepiece, producing the virtual image, III. See also § 95, Ex. 2.

above and § 25, we have the adjacent diagram of rays passing through three lenses: I, II, III.

Surface refraction can be traced in a manner similar to that of § 32 by the formulae

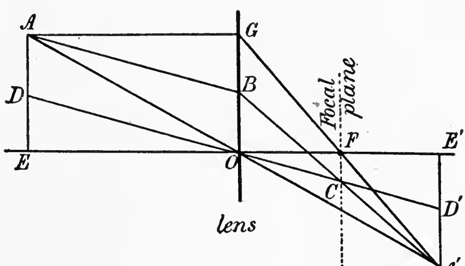
Incident rays	$rs \rightarrow \phi_3 \parallel$
Emergent rays	$rs \rightarrow \phi_2 \parallel$

where $\phi_3 \parallel$ means the point determined by a parallel through the center of curvature and a perpendicular (to the axis) through the focus of the surface, 3 radii (assuming $\mu = \frac{3}{2}$, § 12) from the vertex, measured through the center. $\phi_2 \parallel$ similarly, but 2 radii (§ 13) from the vertex, measured away from the center. (See Appendix, Figs. 1–4.)

Example 1. — Try this method in checking the results of § 72, Exs. 7, 8, 10, 19.

Example 2. — By § 32, drawing a ray from a point in the axis (see ② in preceding diagram), show that in order to get a virtual image (the case of a positive lens used as a microscope) the object must be within the focal distance. (Conf. § 93, Ex. 4.)

34. Parallel Rays meet in the Focal Plane.



By Elementary Geometry, three rays through a point cut off proportional parts on any two parallels; hence

$$\frac{AD = OB}{AE = OG} = \frac{A'D'}{A'E'} \quad \text{or} \quad \frac{OB}{A'D'} = \frac{OG}{A'E'}$$

But by similar triangles, etc.

$$\frac{OC}{CD'} = \frac{OB}{A'D'} = \frac{OG}{A'E'} = \frac{OF}{FE'}$$

Therefore the triangles OCF and $OD'E'$ being similar, C and F are equally distant from the line OG

That is, parallel rays focus in the focal plane (the plane through the focus perpendicular to the axis) at a point determined by the center line; and, conversely, rays from a point in the focal plane emerge parallel, parallel to a line from the point through the center.

35. Standard Formula. — One formula $\left(\text{viz. } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ is used throughout the book, the proper sign (+ or −) being given to the numerical values when used. The use of the two formulae of § 17, one for the positive lens and one for the negative lens, as is the practice of some writers, is apt to lead to confusion, since both formulae apply to both lenses under some conditions. It is the difficulty of distinguishing these conditions that makes the trouble for the non-expert. Hence the decision at the head of this section, since then the only difficulty arises from the selection of the + and − signs. This selection is guided by the rules of the next two sections. (See note to § 17.)

USE OF THE FORMULA $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

(See diagrams of §§ 25–29.)

36. Positive Lens. — Real object, diverging rays, real image (object outside of F):

f and v must have a different sign from u.

Real object, virtual image (object inside of F):

u and v must have a different sign from f.

Converging rays, aerial object:

u, v, and f have the same sign.

For light coming from the left, f is positive (for light from the right, f is negative). (See § 76.)

37. Negative Lens. — Real object, diverging rays:

u, v, and f have the same sign.

Converging rays (converging outside of F), virtual image:

u different sign from v and f.

Converging rays (converging inside of F) real image:

u and v have different sign from f.

For light coming from the left f is negative (for light from the right, f is positive). (See § 76.)

EXAMPLES

Check each calculation by an actual drawing, to scale (§ 38), to avoid large errors; guide the drawing by the formulation of §§ 30, 32, and see §§ 38–41.

Decide on the direction of the ray, thus fixing the sign of u and f (say from the left). If from the left, f will be positive for a positive lens and negative for a negative lens; u will be negative for a real object or an aerial object with diverging rays therefrom, and positive for an aerial object and converging rays. (See § 76.)

From the data given find the other elements.

1. Positive lens with $F = 1$ ft.

(a) $u = -11$ in. $\therefore$ $v = -11$ ft.

(b) $u = -10$ in. $\therefore$ $v = -5$ ft.

(c) $u = -1$ in. $\therefore$ $v = -\frac{1}{11}$ ft.

(d) $u = -20$ ft. $\therefore$ $v = \frac{20}{19}$ ft.

(e) $u = -2$ ft. $\therefore \quad v = 2$ ft.

(f) $u = -1\frac{1}{2}$ ft. $\therefore \quad v = 3$ ft.

2. $u = -2f$ $\therefore \quad v = 2f$

3. $u = -6,\ v = 1$ $\therefore \quad F = \frac{6}{7}$

4. $u = 3$ in., $v = 18$ in. $\therefore \quad F = -3\frac{3}{5}$ in.

5. $u = 12,\ v = 1$ $\therefore \quad F = \frac{12}{11}$

6. $r = 5,\ s = -7$, negative lens, $\mu = \frac{3}{2},\ u = 60$.

Ans. $F = \frac{35}{6},\ v = \frac{420}{79}$, double concave.

7. Positive lens, $r = 7,\ s = 5,\ \mu = \frac{3}{2},\ u = 60$.

Ans. Light from right, $F = -35,\ v = -84$, concavo-convex.

8. Positive lens, $r = -7,\ s = 5,\ \mu = \frac{3}{2},\ u = 60$.

Ans. $F = -\frac{35}{6},\ v = -\frac{84}{13}$, double convex, light from right.

9. Negative lens, $r = 5,\ s = 7,\ \mu = \frac{3}{2}$.

Ans. $f = 15,\ f' = -21,\ F = 35$.

10. Negative lens, $r = 7,\ s = 5,\ \mu = \frac{3}{2}$.

Ans. Light from left, $f = 21,\ f' = -15,\ F = -35$.

11. Negative lens, $r = -7,\ s = -5,\ \mu = \frac{3}{2}$.

Ans. Light from right, $f = -21,\ f' = 15,\ F = 35$.

12. Convex lens, light from right, $F = 5.813$, object 30.56 in front. Where is the image?

Ans. $\dfrac{1}{v} = \dfrac{1}{30.56} + \dfrac{1}{-5.813} = 0.03273 - 0.1720 = -0.1392$

$= \dfrac{-1}{7.183}. \quad \therefore v = -7.183$ to left.

13. A telescope has a field glass of $23\frac{3}{8}$ inches focus and an erecting eyepiece composed of 4 lenses as follows, reading towards the eyepiece, 2, $1\frac{7}{8}$, $1\frac{7}{8}$, $1\frac{3}{8}$ inch focus, with the separations $2\frac{1}{4}$, 4, 2 inches. To trace the conjugate foci.

Since the last four lenses are fixed and the focussing is done by adjusting the combination relative to the field glass, we take as the starting point the virtual image seen by the eye. This will be seen at a distance determined by the "set of the eye" of the observer. (See § 109.)

We assume the "far set" eye and the rays to the virtual image parallel. This makes the object for the fourth lens (counting from the left) at the focus of that lens; and indicating by v_1, u_1 the conjugate distances for the first lens, etc., we have the following series of conjugate distances.

$$v_4 = \infty,\ u_4 = -\frac{11}{8}$$

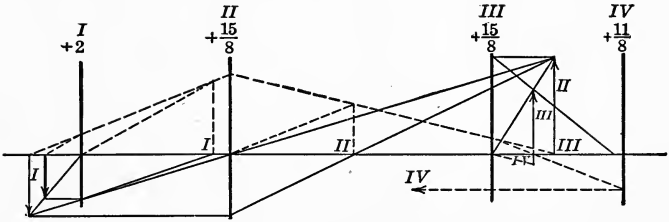
$$v_3 = 2 - \frac{11}{8} = .625. \qquad \frac{8}{5} - \frac{1}{u_3} = \frac{8}{15}. \quad \therefore\ u_3 = .937$$

$$v_2 = 4 + .937 = 4.937. \quad \frac{1}{4.937} - \frac{1}{u_2} = \frac{8}{15}. \quad \therefore\ u_2 = -3.012$$

$$v_1 = -3.012 + 2.25 = -.762. \quad \frac{1}{-.762} - \frac{1}{u_1} = \frac{4}{9}$$

$$\therefore\ u_1 = -.552$$

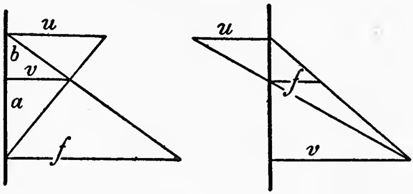
This shows that the focus of the field glass should be .552 inches in front of the first lens. This is approximate only, since the real lenses must be treated as thick lenses, as in Chapter III.



The lenses, with their corresponding images and foci, are designated by Roman numerals. The dotted lines show the course of a ray from the foot of the aerial object, with the construction lines (§ 35). Notice how the inverted object (aerial) is converted into an erect one by the second lens. The virtual image being at infinity, the last course of the ray is horizontal, as shown.

GRAPHIC CHECK ON CALCULATIONS

38. Inspection of the diagrammatic constructions will show that they fall under one or the other of the following diagrams, or modifications of these.



Where all the quantities are on one side of the zero line (lens line), as in the left-hand diagram, we have (by similar triangles):

$$\frac{a}{a+b} = \frac{v}{u}, \quad \frac{b}{a+b} = \frac{v}{f}, \text{ whence } \frac{v}{u} + \frac{v}{f} = \frac{a+b}{a+b} = 1$$

or

$$\frac{1}{u} + \frac{1}{f} = \frac{1}{v}$$

which is the same as in § 16.

Worded, this becomes, *The reciprocal of the mid line = the sum of the reciprocals of the end lines* (when all the quantities are on the same side of the zero line).

39. Similarly for the right-hand diagram, unless we take into account the signs of the quantities, in which case:

Where two of the quantities are on different sides of the zero (lens) line (right-hand diagram) *The reciprocal of the mid line = the reciprocal of the end line on the same side of the zero line as the mid line — the reciprocal of the end line on the other side of the zero line.*

USE OF THE DIAGRAMS. — *To insure accuracy in signs and to detect material (large) errors, plot these diagrams to scale. Lay off the end lines any distance apart, draw the diagonals and see if the mid line fits in size and sign. Or lay off* f *and* u *(any distance apart) and then by means of the two diagonals determine* v.

EXAMPLES

Ex. 1. $\frac{1}{-11} = \frac{1}{-11\cdot 12} - \frac{1}{12}.$ Ex. 2. $\frac{1}{-10} = \frac{1}{-5\cdot 12} - \frac{1}{12}.$

Ex. 3. $\frac{1}{1} = \frac{1}{2} - \frac{1}{-2}.$ Ex. 4. $\frac{1}{1} = \frac{1}{3} - \frac{1}{-\frac{3}{2}}.$

Ex. 5. $\frac{1}{\frac{6}{7}} = \frac{1}{1} - \frac{1}{-6}.$ Ex. 6. $\frac{1}{18} = \frac{1}{3} + \frac{1}{-\frac{18}{5}}.$

Ex. 7. $\frac{1}{1} = \frac{1}{12} + \frac{11}{12}.$

GRAPHIC CHECK ON CALCULATIONS

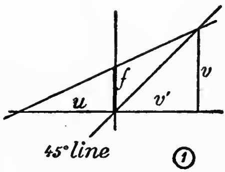
This method is given in some detail because so many books use one or the other of the diagrams.

40. Pos. lens with + f. By similar triangles

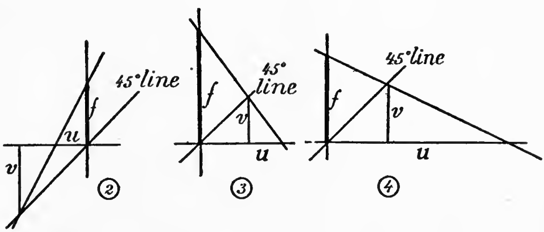
$$\frac{u}{f} = \frac{u + v'}{v} \quad \text{or} \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \qquad [v = v'$$

which is the same as the equation of § 17 for a positive lens.

Use of the Diagram. — Set off f and u in proper size and direction, draw the line through their ends: its intersection with the 45° line will give v both in size and sense. u on the right indicates an aerial image made by some preceding lens.



Variations of this are, as the object moves from the left, being aerial when on the right of the lens, the rays of light coming from the left,



Diagrams showing the relative sizes and positions of u and v.

① Real object beyond the focal distance, real image, inverted. (Light from the left.)

② Real object within the focal distance, virtual image, erect.

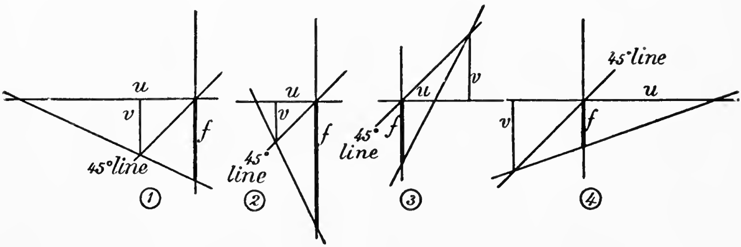
③ Aerial object within the focal distance, real image, erect.

④ Aerial object beyond the focal distance, real image, erect.

41. Negative lens with − f. In the same manner, we get the following diagrams:

Use of the Diagrams. (See § 40.)

Notice. — When u and v have the same sign, the image is erect. When u and v have a different sign, the image is inverted. (Both lenses.)



(1) Real object beyond the focal distance, virtual image, erect.
(2) Real object within the focal distance, virtual image, erect.
(3) Aerial object within the focal distance, real erect image.
(4) Aerial object beyond the focal distance, virtual inverted image.

POWERS OF LENSES: DIOPTERS

42. In the expression $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, $\frac{1}{f}$ is called the *power* of the lens (to alter a light-wave front), and when v and u are expressed in meters (or its practical equivalent, 40 inches), the power units are called *diopters*.

Similarly, $\frac{1}{v} = p$ and $\frac{1}{u} = p'$ are called, for convenience of reference, *powers of the distances.*

If we call the power of the lens D, then

$$D = p - p' = (\mu - 1)(q - q') \qquad \left[q = \frac{1}{r}, q' = \frac{1}{s}\right.$$

Dioptric units are generally used by opticians in connection with (thin) spectacle lenses. The power of a combination of lenses equals the sum of the powers. (See § 75.)

EXAMPLES

43. Ex. 1. $u = -80$ cm., $f = 20$ cm., $v = ?$ (Light from left.)

Ans. $p = \frac{1}{v} = D + p' = \frac{100}{20} + \frac{100}{-80} = 5 - 1.25 = 3.75.$

$\therefore v = \frac{100}{3.75} = 26\frac{2}{3}$ cm.

Ex. 2. $u = -20$ in., $v = 5$ in., $f = ?$

Ans. $D = \frac{40}{5} - \frac{40}{-20} = 8 + 2 = 10.$ $f = \frac{40}{10} = 4$ in.

Ex. 3. $f = 10$, $u = -8$, $v = ?$ (Positive lens.)

Ans. $D = \frac{40}{10} = p - \frac{40}{-8} = 4 = p + 5.$ $\therefore p = -1.$

Ex. 4. Four lenses in contact: (*a*) a plane concave of 4 diopters; (*b*) a positive meniscus of $r = 2$ in., $s = 5$ in.; (*c*) a biconvex of 50 cm. focus; (*d*) a biconcave of $33\frac{1}{3}$ cm. focus. What is the focus of the combination? (Light from left.)

Ans. $(a) = -4\ D.$ $(b) = \frac{1}{2}\left(\frac{40}{2} - \frac{40}{s}\right) = 6\ D.$

$(c) = \frac{100}{50} = 2\ D.$ $(d) = -\frac{100}{33\frac{1}{3}} = -3\ D.$

Therefore, combination $= -4\ D + 6\ D + 2\ D - 3\ D = D$, and the result is a positive lens of 100 cm. focus, projecting a real inverted image. (See § 25.)

44. The focal length expressed in inches gives the *number of the lens.* (Obsolete.)

45. Spectacles for Farsighted. — Positive lens, virtual image.

The formula of § 16 becomes:

$$\frac{1}{-v} - \frac{1}{-10} = \frac{1}{+f}$$

the *10* and v being taken negative, because we want a virtual image. See diagram of § 27. v must be the *nearest* distance at which the wearer can conveniently see without spectacles, *10* being the distance at which he holds the book. The image is virtual.

46. Spectacles for Nearsighted. — Negative lens.

In this case the formula of § 16 becomes:

$$\frac{1}{-v} - \frac{1}{-u} = \frac{1}{-f}$$

u, v, and f have the same sign, § 37. v must be the *greatest* distance at which the wearer can see clearly without spectacles, *10* inches being the distance at which the book is held. The image is virtual. If f is less than v, he can see objects at all distances over *10* inches, since the virtual image is always within his visual distance. See first diagram of § 29.

EXAMPLES

1. If longest distance for distinct vision is 15 cm., what lens will enable the wearer to see all distant objects? *Ans.* 15 cm. or under.

2. Book is held at 1 ft. with concave 6 in. focal lens. Where is the image? *Ans.* 4 in.

3. A man can read distinctly at 15 cm. What lens must he use if he wants to read easily at 60 cm.? *Ans.* $f = -20$.

4. If the nearest distance for distinct vision is 15 inches, what focal length of spectacle is required if the book is held at 10 inches? *Ans.* 30 in.

5. If the shortest distance for distinct vision is 1 m., what length spectacle is wanted for object at 25 cm.?

Ans. $\frac{1}{f} = \frac{1}{.25} - 1 = 4 - 1 = 3$ diopters, or $f = 33\frac{1}{3}$ cm.

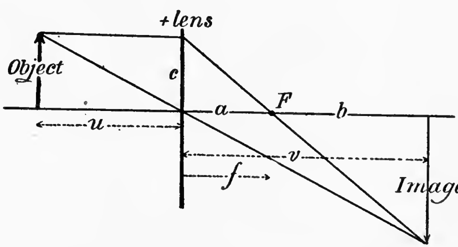
6. Best vision at 3 ft. With 1 ft. spectacles the book should be held at . . .? *Ans.* 9 in.

7. A longsighted person with + glasses of 40 cm. length finds he must hold the book no nearer than 30 cm. for comfort. What is his nearest point of distinct vision? *Ans.* 120 cm.

8. A longsighted person can only see distinctly at 48 cm. or more. By how much will he increase his range of vision with convex spectacles of 32 cm. focus? *Ans.* 48 − 19.2 = 28.8.

9. A person whose distance of most distinct vision is 20 cm., uses a reading glass of 5 cm. focus. How far from the book must it be held? *Ans.* 4 cm.

47. Magnification for Convex Lens, Real Image (Camera, etc.).



Since $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, or $\frac{1}{\mathbf{v}} + \frac{1}{\mathbf{u}} = \frac{1}{\mathbf{f}}$ for positive lenses,

$\frac{\mathbf{v}}{\mathbf{u}} = \mathbf{M}$ = magnification factor = ratio of image to object

= ratio of image distance to object distance

$$= \frac{\mathbf{v}}{\mathbf{f}} - 1 = \frac{\mathbf{f}}{\mathbf{u} - \mathbf{f}}$$

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}} = \frac{\text{image}}{c} = \frac{b}{a} = \frac{\mathbf{v} - \mathbf{f}}{\mathbf{f}}$$

$$\mathbf{u} = \frac{\mathbf{f}}{\mathbf{M}} + \mathbf{f} = \text{distance to object}$$

$$\mathbf{v} = \mathbf{fM} + \mathbf{f} = \text{distance to image}$$

If $\mathbf{M} = 1$, then $\mathbf{u} = 2f$, $\mathbf{v} = 2f$.

If $\mathbf{M} = -1$, which is equivalent to saying, image same size but not inverted, then $\mathbf{u} = 0$, $\mathbf{v} = 0$.

Note. — Ordinarily (*when we are taking account of direction*) $+M$ indicates erect image; $-M$, inverted image. Do not get the two cases confused.

EXAMPLES

1. In Ex. 9, § 46, what is the magnifying power?

Ans. $\dfrac{5}{5-4} = 5.$

2. An engraver uses a 4 in. focus magnifying glass, close to the eye. Where must he hold his work to get a magnification of 4? *Ans.* $\dfrac{4}{4-u} = 4. \quad \therefore u = 3.$

3. An object is 3 ft. in front of a 6 in. lens. What is the magnification? *Ans.* $\frac{1}{2}/(3 - \frac{1}{2}) = \frac{1}{5}$.

4. $v = 8$, 4-inch lens. *Ans.* $M = 1, u = 8$.

5. $v = 12$, 4-inch lens. *Ans.* $M = 2, u = 6$.

6. $v = 16$, 4-inch lens. *Ans.* $M = 3, u = 5\frac{1}{3}$.

7. $v = 20$, 4-inch lens. *Ans.* $M = 4$, $u = 5$.

8. An object 8 cm. high is 1 m. from an equiconvex lens of index of refraction 1.5 and radius of curv. 0.4 m. Where is the image and what is its size? *Ans.* Image $\frac{2}{3}$ m. on other side and of height $5\frac{1}{3}$ cm.

9. In Ex. 1, § 37, what are the magnifications? *Ans.* 12, 6, $\frac{1}{11}$, $\frac{1}{19}$, 1, 2.

10. Converging lens, with object 5 in. from lens. Image = 6 times the object. Where is the image and what is the focal length? *Ans.* $v = 30$ in. $f = \frac{30}{7}$.

11. Required an image of 3 mag. by lens of F focal length. How far must the screen be from the object?

Ans. $\frac{4}{3}F$. $\quad \frac{1}{x} + \frac{1}{3x} = \frac{1}{F}. \quad \therefore x = \frac{4}{3}F.$

12. An object is a distance d from a screen, and a thin pos. lens is placed to form an image. If the lens be moved a distance $p = \sqrt{d^2 - 4df}$, another image will be formed whose linear dimensions are to those of the former as $(d - p)^2$ to $(d + p)^2$.

13. A disc 1 inch in diameter, 8 inches from the eye, is seen through a convex lens of 8 inches focus, placed halfway between. What should be the diameter of the lens to see the whole of the disc at once? What is the distance of the image from the eye? *Ans.* Diam. of lens = $\frac{2}{3}$ in. Dist. of image from eye = 12 in.

14. A concave lens is blackened except a circle of 4 cm. diameter at the center. A beam of sunlight through this gave an illuminated circle of 20 cm. diameter on a screen 64 cm. from the lens. Show that the focus of the lens is -16 cm. Use the first diagram of § 29.

15. If $\mathbf{u} = 2f$, then $\mathbf{v} = 2f$, $M = 1$.

16. If u is very large, show that $\frac{M}{M'} = \frac{f}{u-f} \cdot \frac{u-f'}{f'} = \frac{f}{f'}$ practically, and therefore that for distant objects the sizes of the image are proportional to the focal radii of the lenses.

48. Copying, Enlarging, etc., with the Camera.

To find distance to plate, etc., for given magnification or reduction.

$$v = \text{distance from lens to screen (plate)}$$
$$= \text{camera extension.}$$
$$= F(M+1) = \frac{F}{N} + F \quad (\S\, 47)$$

$$\left[\begin{array}{l} M = \text{magnification factor} \\ N = \text{reduction factor; } M = \frac{1}{N} \\ F = \text{focal length of the lens} \end{array}\right.$$

$$u = \text{distance from lens to object}$$
$$= F + \frac{F}{M} = F + NF$$

Strictly these distances should be measured from the nodal points (see § 63), but approximate values and measurements are sufficient for a first adjustment, the final being made by trial.

EXAMPLES

1. 6 in. lens, 12 in. drawing, 4 in. copy, whence $N = 3$.

Ans. $v = \frac{6}{3} + 6 = 8$, $u = 3 \cdot 6 + 6 = 24$.

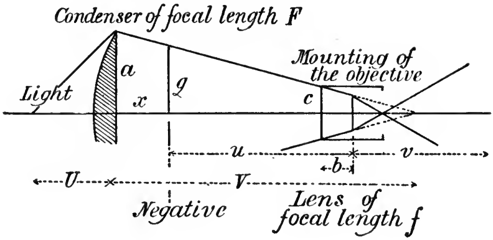
2. 6 in. lens, 4 in. plate, 12 in. copy, whence $M = 3$.

Ans. $v = 24$, $u = 8$.

3. A candle stands a yard from the screen. What lens and where must be used to get an image 5 times as large?

Ans. 5 in. lens, 30 in. from the screen.

49. Best Position for Condenser.



$$\frac{1}{V} - \frac{1}{U} = \frac{1}{F}$$ [§ 16

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{1}{\mathbf{v}} + \frac{1}{\mathbf{u}}$$

$$\frac{\mathbf{v}}{\mathbf{u}} = \mathbf{M} = \text{Magnification}$$

$$\mathbf{v} = (\mathbf{1} + \mathbf{M})\,\mathbf{f}$$ [§ 48

$$\mathbf{u} = \frac{\mathbf{1} + \mathbf{M}}{\mathbf{M}}\,\mathbf{f}$$

To get all the picture within the cone of rays,

$$\frac{a}{g} = \frac{V}{V - x}$$

whence x can be calculated.

To get the cone of light to just fill the mounting,

$$\frac{a}{c} = \frac{V}{V - x - u + b}$$

whence u can be found.

For sunlight, $V = F$, $(U = \infty)$, whence

$$F = \frac{a}{g - c}(1 + M)\,f$$

If $x = 0$, $\dfrac{1}{U} = \dfrac{a - c}{a} \cdot \dfrac{1}{(1 + M)f + b} - \dfrac{1}{F}$, whence U can be found.

Example. — $F = 3$, $f = 6$, $M = 5$, $a = \frac{3}{2}$, $c = \frac{1}{2}$, $b = \frac{3}{2}$.

$$\therefore \frac{1}{U} = \frac{\frac{3}{2} - \frac{1}{2}}{\frac{3}{2}} \cdot \frac{1}{36 + \frac{3}{2}}. \quad \therefore U = 3.17$$

50. Exposure.

T = time of exposure for distance v of image, with aperture d

$$= \left(\frac{v}{v_1'}\right)^2 \cdot \left(\frac{d_1}{d}\right)^2 \cdot T_1$$

(the subscripts indicating the corresponding quantities for some known exposure with a satisfactory result)

$$= \left(\frac{FM + F}{F_1M_1 + F_1}\right)^2 \cdot \left(\frac{F_1}{f_1} \cdot \frac{f}{F}\right)^2 \cdot T_1 = \left(\frac{M + 1}{M_1 + 1}\right)^2 \cdot \left(\frac{f}{f_1}\right)^2 \cdot T_1$$

$$= \left(\frac{M + 1}{M_1 + 1}\right)^2 \frac{U.S.}{U.S._1} T_1$$

$$\left[\begin{array}{rl} U.S. &= U.S. \text{ numbers} \\ f, f_1 &= \text{the } f \text{ numbers; see § 51} \\ M, M_1 &= \text{magnification} \end{array}\right.$$

In use, disregard the letters in which there has been no change of conditions, and see note at the end of Example 1.

EXAMPLES

1. With an 8 in. lens, with $f/20$ enlarging 5 times, 40 seconds exposure was required. What exposure is required for a 9 in. lens $f/30$ and enlarging 6 times?

$$\textit{Ans.} \quad T = \left(\frac{30}{20} \cdot \frac{7}{6}\right)^2 40 = 122 \text{ sec.}$$

Notice that the f number determines the exposure without regard to the focal length. E.g. $f/20$ requires the same time whatever the lens.

51. Slowness factor $= \dfrac{\text{focal length}}{\text{aperture diameter}} = n$, written $=$ f/n and called the f number.

Time of exposure varies as the square of the slowness factor. For example, a $\dfrac{f}{13}$ lens requires $\dfrac{169}{64}$ the exposure of a $\dfrac{f}{8}$ lens. $\quad \dfrac{169}{64} = \dfrac{13^2}{8^2}$

The *U.S.* numbers give the relative time of exposure,

$$\frac{f}{16} = U.S.\ 16$$

whence we can find corresponding f and *U.S.* numbers by the formulae

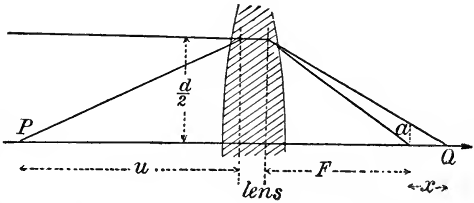
$$f \text{ number} = 4\ \sqrt{U.S. \text{ number}}$$

$$U.S. \text{ number} = \left(\frac{f \text{ number}}{4}\right)^2$$

52. Hyperfocal Distance. — When the object is too near to the lens, what produces confusion in the picture is the overlapping of images. It is found that a slight overlapping is not distinguishable. This occurs when the two images of a given point are not more than $\frac{1}{100}$ inch apart. Hence up to the point in front of the lens where the separation of images is not more than $\frac{1}{100}$ inch, all objects will apparently be in focus.

The distance of this point from the lens is called the

hyperfocal distance, the distance to the nearest distinct object beyond which all objects are apparently in focus.



The different images of a given object are evidently scattered over a circle. The largest circle permissible without confusion is called the *circle of confusion.*

F = focal length in inches
P = nearest distinct object
Q = image of P
a = radius of circle of confusion
d = diam. of stop in inches

$$f = \frac{F}{d} = f \text{ number}$$

$$a = \frac{1}{200} \text{ in.}$$

By sim. triangles
$$\frac{x}{a} = \frac{F + x}{d/2}$$

But § 17
$$\frac{1}{F + x} + \frac{1}{u} = \frac{1}{F}$$

$$\frac{1}{u} = \frac{1}{F} - \frac{1}{F + x} = \frac{x}{F\,(F + x)}$$

$$u = F\,\frac{F + x}{x} = \frac{F\,\frac{d}{2}}{a} = \frac{Fd}{2\,a}$$

$$= \frac{\text{focal length} \times \text{diam. of stop}}{\text{diam. of circle of confusion}}$$

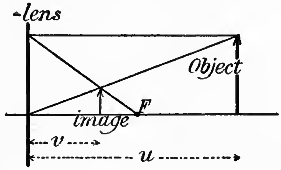
$$= \frac{F^2}{f\,\frac{1}{100}}$$

$$= \frac{100\,F^2}{f} \text{ in.} = \frac{100\,F^2}{12\,f} \text{ ft.}$$

= distance to nearest distinct object, beyond which all objects are apparently in focus

Note. — The diagram is drawn for a thick lens. If the two vertical dotted lines should be brought together, it would make the proper diagram for a thin lens, with no change in the mathematics.

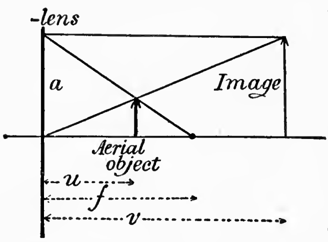
53. Magnification and Reduction for Negative Lens.



$$\text{Reduction} = \frac{1}{N} = \frac{\text{image}}{\text{object}} = \frac{f-v}{f} = 1 - \frac{v}{f}$$

$$= 1 - \frac{u}{f+u} = \frac{f}{f+u}$$

As the object is moved nearer the lens, the image grows larger, until with the object at the lens we get unit reduction.



54. If converging rays (due to convex lens) are coming from the left, we have the following diagram, the real image of the convex lens being the aerial object of the negative lens.

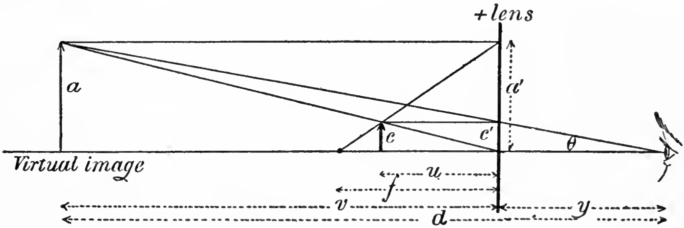
$\frac{\text{Image}}{\text{Object}}$ = magnification of image of the + lens, due to the − lens.

$$= \frac{a}{\text{aerial object}} = \frac{f}{f-u} = \frac{f+v}{f} = 1 + \frac{v}{f}$$

This is the telephoto combination spoken of in Chapter V. v is there called the bellows extension and denoted by E,

whence $\quad M = 1 + \frac{E}{f} \quad$ and $\quad E = f\,(M - 1)$

55. Magnifying Power of a Positive Lens used as a Microscope.



In this case the image seen by the eye is virtual and subtends the angle θ shown in the diagram. Since this angle is very small, it is measured by its tangent, and

$$\theta = \tan\theta = \frac{a}{d}$$

The angle subtended by the object is

$$\phi = \frac{c}{\mathrm{u} + \mathrm{y}}$$

and the ratio of these is the

$$\text{Apparent magnification} = \frac{\theta}{\phi} = \frac{a\,(\mathrm{u} + \mathrm{y})}{cd}$$

$$= \frac{cf}{\mathrm{f} - \mathrm{u}} \cdot \frac{\mathrm{u} + \mathrm{y}}{cd}$$

$$= \frac{\mathrm{f}\,(\mathrm{v} + \mathrm{y}) + \mathrm{vy}}{fd}$$

$$= \frac{\mathrm{f}d + \mathrm{vy}}{\mathrm{fd}} = 1 + \frac{\mathrm{vy}}{\mathrm{fd}}$$

$$\left[\frac{a'}{c} = \frac{f}{f - u}\right.$$

$$\mathrm{u} = \frac{\mathrm{fv}}{\mathrm{f} + \mathrm{v}},$$

$$\left.\frac{1}{-\mathrm{v}} - \frac{1}{-\mathrm{u}} = \frac{1}{\mathrm{f}} \quad \S\,36\right.$$

$$= 1 + \frac{(d - y)\,y}{fd}$$

This is a maximum $= 1 + \frac{d}{4f}$, when $y = \frac{1}{2}d$, which explains why some readers like to push their spectacles down towards the end of the nose.

The distance d, the distance for distinct vision, is generally taken as 10 inches (25 cm.), but should be taken a specific value for each observer.

56. Ordinarily the conventional magnification of the lens is stated as the ratio of the actual size of the image to that of the object, viz.

$$\text{Conventional magnification} = \frac{a}{c} = \frac{v}{u}$$

$$= \frac{v\,(v + f)}{vf} \qquad \left[u = \frac{fv}{f + v}\right.$$

$$= 1 + \frac{v}{f} = \frac{f}{f - u}$$

EXAMPLES

1. A convex lens, of focal length $\frac{1}{5}$ inch, is used by a 14 inch (nearest distance for distinct vision) eye. What is the magnification?

Ans. Mag. $= 14/\frac{1}{5} + 1 = 71$.

2. A 2 yard eye uses a 2 ft. lens. How far from the glass should the object be placed?

Ans. $\frac{1}{6} - \frac{1}{u} = \frac{1}{-2}$; therefore $u = \frac{3}{2}$ ft.

3. Check Ex. 2 by comparison of images, graphic construction, § 32.

CHAPTER III

THICK LENSES

57. One method of finding the equivalent focus of a thick lens is to select a thin lens (spectacle lens) which will give on the ground glass an image of *exactly the same size* as the thick lens gives, the object being very distant. The focal length of this thin lens will be the focal length of the lens under consideration. On the mounting of the thick lens mark off this distance from the ground glass when in focus. This point is called the *principal point of emergence.*

Turning the lens around, we get a similar point for the other end of the lens. These two separated points mark the points from which evidently measurements for focal radii are to be made, and correspond to the optical center of a thin lens.

Like optical centers, these principal points will be found to be points around which the lens can be twisted (about a vertical axis) without affecting the image on the ground glass.

58. Suppose an object to take the successive positions a, b, c, and then the aerial objects at d and e, with the resulting images a', b' . . .

At the optical center of the thin lens, and only there (Conf. § 47), will the object and image have the same size and sense (image not inverted).

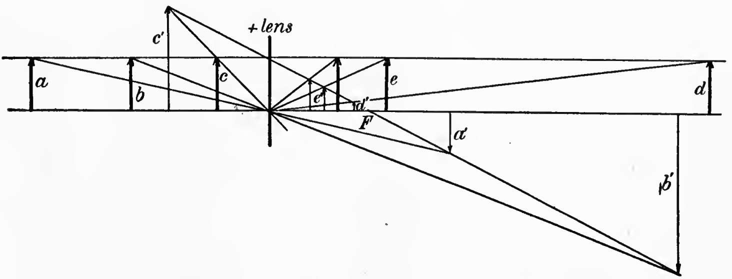


Diagram showing the images resulting from successive positions of the object, and the resulting changes in size of the image; i.e. its magnification with the corresponding images a', b', c'. . .

59. In § 32 we found that revolution about an axis through the optical center, the point from which the focal radius is measured, did not disturb the image. Experiment shows that revolution about an axis through the principal point of emergence, the point from which we measure the focal distance, does not disturb the image of a distant object. This is the point around which panoramic cameras are revolved.

60. Principal Points. — Just as image and object have the same size and relation at the optical center of a thin lens, so we might anticipate that for a thick lens the image and object would have the same size and relation at the principal points, the points from which the radii are measured. (Conf. § 69.)

61. We can find these principal points as follows:

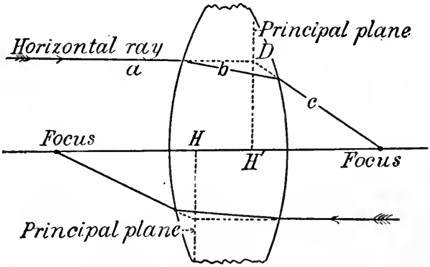


Diagram showing construction for the determination of the principal points. The order of the letters indicates the order of construction. *b*, *c*, found by § 7. The upper half gives the construction for one principal point, the lower half for the other.

Caution. — This construction applies strictly only to points near the axis, but it serves to illustrate the principle for future use.

Theoretically we could, by picturing the surfaces as straight lines, get a correct graphic construction, but the disparity between the thickness of the lens and the radii is generally so great that the graphic construction is of little value by reason of its acute intersections.

62. Since there are two points around which we can

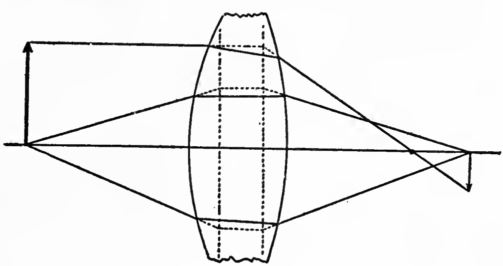


Diagram illustrating the apparent horizontal transference between the principal planes.

revolve the lens without effect on the image (the lens being reversed so as to make each one a point of emergence), i.e. two points like the optical center of the thin lens, two points where the object and image have the same size and relation, as shown by the diagram (Conf. the diagram of § 61), the effect is as if the rays from the object passed to the first principal plane and then were transferred horizontally to the other principal plane so as to keep the object and image the same size from plane to plane. (Conf. § 105 after reading § 64.)

This equivalent pair of parallel surfaces is called the *equivalent thin-split.*

63. Nodal Points. — The following construction gives two new points of importance called *nodal points.*

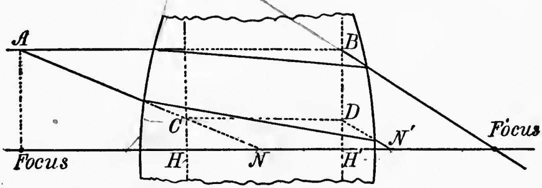


Diagram illustrating the location of the nodal points.

From A, a point in one focal plane, draw the horizontal ray AB, which is of course refracted to the focus F'. Draw AC parallel to BF'. By the property of principal planes C is carried to D, and by the property of rays from a point in a focal plane (see § 35) DN' is parallel to BF'.

N and N' are two points, nodal points, which have the property that incident rays through one of them (e.g. N) emerge parallel through the other (e.g. N'). In this they resemble the optical center of thin lenses. (Conf. § 70.)

64. Evidently $HN = H'N' = F'H' - FH$.

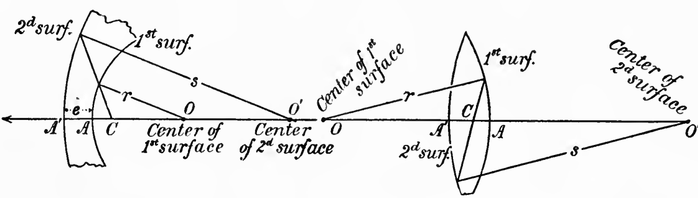
[Equal triangles, etc.

Therefore if $FH = F'H'$, i.e. the focal distances the same, due to same media on both sides of the lens, the usual case (but Conf. § 71), then H and N coincide, likewise H' and N'.

Every incident ray through the first nodal point emerges as a parallel ray through the second nodal point. Therefore the angle subtended by an object at the first nodal point equals the angle subtended by the image at the second nodal point, just as in the thin lens the angles subtended at the center by object and image are the same size.

In the human eye, the second nodal point is within the crystalline lens about .4 mm. from the back. (Conf. § 71.)

65. Optical Center.



Construction. — From the centers of the two surfaces draw parallel rays and find the point C as shown.

By sim. triangles $\dfrac{CO}{CO'} = \dfrac{r}{s}$ (left-hand diag.)

$$\frac{-CO}{CO'} = \frac{-r}{s} \text{ (right-hand diag.)}$$

$$\therefore \quad CO = \frac{r}{s-r}\overline{OO'} = \frac{r}{s-r}(s-r-\text{e})$$

$$= r - \frac{\text{e}r}{s-r}$$

EXAMPLES

1. Neg. lens. $r = 2, s = -3, e = 1.$

$$Ans.\quad AC = \frac{-2}{5},\ A'C = \frac{3}{5}$$

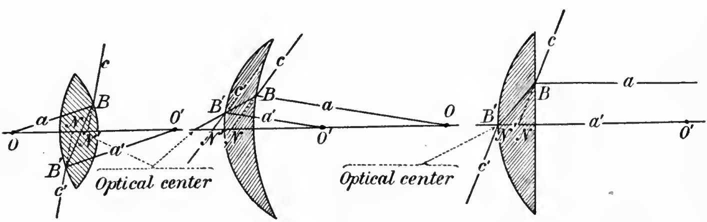
2. Pos. lens. $r = -3, s = 2, e = 1.$

$$Ans.\quad AC = \frac{-3}{5},\ A'C = \frac{2}{5}$$

66. Construction for Nodal Points.

Caution. — These constructions apply strictly only to points near the axis, but they serve to illustrate the principle. (Conf. remark, § 61.)

Draw a and a' parallel through the centers O, O', giving the points B, B'. From the ray BB' construct the refracted rays c, c'. Where c, c' prolonged cut the axis will



be the nodal points N, N'. Where BB' cuts the axis is the optical center, since c is parallel to c', being equally inclined to the parallel radii, a, a'.

$$\therefore\quad AC = AO - CO = \frac{\mathbf{e}r}{s - r} = \frac{-\mathbf{e}f}{f' + f}$$

$$\therefore\quad A'C = AA' + AC$$

$$= \frac{\mathbf{e}s}{s - r} = \frac{\mathbf{e}f'}{f' + f},\ \text{See diag. § 65}$$

$$f = \frac{\mu r}{\mu - 1}$$

$$f' = \frac{-\mu s}{\mu - 1}$$

$$\mathbf{e} = AA'$$

Hence the position of the optical center is fixed for two given surfaces a distance e apart, since only constants enter into its value.

If the light comes from the left, A and A' interchange places, also r and s, and $A'C = \dfrac{-\mathrm{e}f'}{f+f'}$, $AC = \dfrac{\mathrm{e}f}{f'+f}$

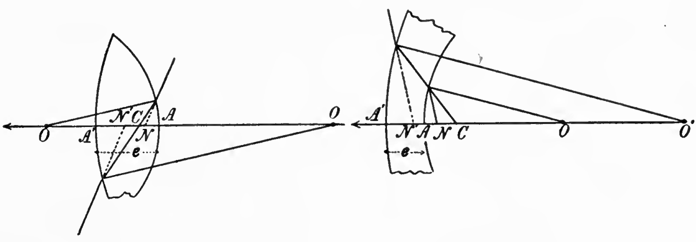
67. Calculation for Nodal Points.

Evidently any ray passing (after refraction) through C will enter and emerge in parallel lines, since the surfaces at the points of incidence and emergence are parallel (being perpendicular to the parallel radii from O, O').

Evidently a ray pointing to N before refraction will after refraction emerge in a parallel direction as if coming from N'.

N and C are conjugate foci for the first surface, therefore, § 11,

[**Note.** — Use the diagram on the right (for reasons given in § 21, note) until second reading.]



$$\frac{1}{AN} = \frac{\mu}{AC} - \frac{\mu - 1}{r} = -\mu\frac{f+f'}{\mathrm{e}f} - \frac{\mu}{f} \qquad \text{[§ 66}$$

$$= -\mu\frac{f+f'+\mathrm{e}}{\mathrm{e}f}$$

$$\left(= \mu\frac{f+f'-\mathrm{e}}{\mathrm{e}f}, \text{ if light comes from the left}\right)$$

Therefore AN = dist. from first vertex to corresponding nodal point

$$= \frac{-\mathrm{e}f}{\mu\,(f + f' + \mathrm{e})}$$

$$\left(= \frac{\mathrm{e}f}{\mu\,(f + f' - \mathrm{e})}, \text{ if light comes from left}\right)$$

Similarly $A'N'$ = distance from second vertex to corresponding nodal point

$$= \frac{\mathrm{e}f'}{\mu\,(f + f' + \mathrm{e})}$$

$$\left(= \frac{-\mathrm{e}f'}{\mu\,(f + f' - \mathrm{e})}, \text{ if light comes from the left}\right)$$

NN' = distance between the nodals

$$= \mathrm{e} - \frac{\mathrm{e}f + \mathrm{e}f'}{\mu\,(f + f' + \mathrm{e})} = \frac{(\mu - 1)\,(f + f') + \mu\mathrm{e}}{\mu\,(f + f' + \mathrm{e})}$$

$$\left[= \frac{(\mu - 1)\,(f + f') - \mu\mathrm{e}}{\mu\,(f + f' - \mathrm{e})}, \text{ if light is from the left}\right.$$

$$= \frac{\mathrm{e}\,(r - s + e)\,(\mu - 1)}{\mu\,(r - s + e) - \mathrm{e}}$$

$$\left[= \frac{e\,(r - s - \mathrm{e})\,(\mu - 1)}{\mu\,(r - s - \mathrm{e}) + \mathrm{e}}, \text{ if light is from the left}\right.$$

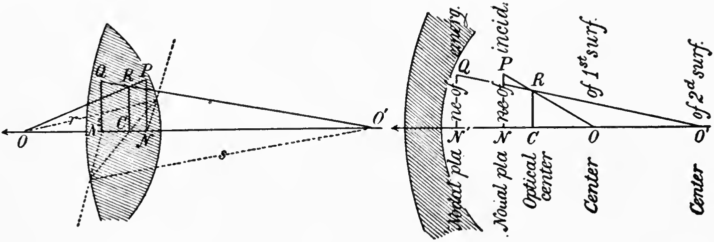
$$= \frac{\mu - 1}{\mu}\,\mathrm{e} \text{ (neglecting very small terms)}$$

$$= \frac{1}{3}\,\mathrm{e} \text{ for glass}$$

68. In computation check the numerical value for NN' by the separate values, $e - AN - A'N'$, and check by graphic construction, as in § 73.

Ex. 1. Show that $AN : A'N'$ has the ratio between **r** and **s**, the radii of the surfaces.

69. Image in One Nodal Plane of Object in the Other. If P is the object in one nodal plane (which may be outside the prism altogether, see § 72, Ex. 3), we can find its image in the other nodal plane by tracing known rays.



The explanatory details of one diagram apply equally to the other.

The known rays are rays through the center of curvature, which enter the corresponding surface without refraction.

The ray PO, which is unrefracted by the first surface, gives an image R in the plane RC. The image R becomes the object for a new image Q, made in the nodal plane N' by the unrefracted ray $O'R$.

$$\frac{PN}{RC} = \frac{NO}{CO}, \quad \frac{QN'}{RC} = \frac{N'O'}{CO'} \qquad \text{[Sim. triangles}$$

Therefore

$$\frac{PN}{QN'} = \frac{PN}{RC} \cdot \frac{RC}{QN'} = \frac{NO}{CO} \cdot \frac{CO'}{N'O'} = \frac{NO}{N'O'} \cdot \frac{CO'}{CO}$$

[Sim. triangles

$$= \frac{r}{s} \cdot \frac{s}{r} = 1 \qquad \left[\frac{CO}{CO'} = \frac{r}{s}, \; \frac{ON}{O'N'} = \frac{r}{s} \quad \text{Sim. triangles}\right.$$

Therefore

$$PN = QN'$$

Hence the object in one nodal plane has an equal and erect image in the other nodal plane; i.e. all rays passing through P in one plane will pass through Q in the other. PQ is parallel to OO'. The image in one nodal plane is transferred horizontally without change of size to the other nodal plane. (Conf. § 60.)

70. Lens separating Different Media.

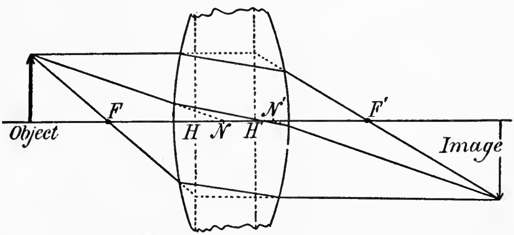
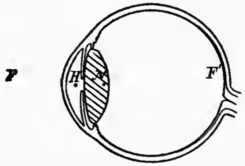


Diagram showing the paths of two sets of rays when the principal points and the nodal points do not coincide. Compare this with the diagram of § 74, where the principal points and the nodals coincide.

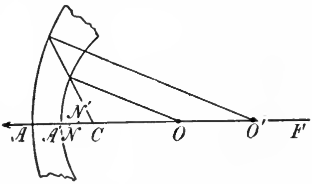
71. The human eye illustrates this case, the aqueous humor being on one side of the lens and the vitreous humor on the other, under which circumstances the principal points and the nodal points are separated and the two foci are different. (Conf. § 22.)



The principal points are very close together at H, about 2 mm. behind the cornea; the nodal points almost as close together at N, about 7 mm. behind the cornea.

The anterior focus is at F, about 13.7 mm. in front of the cornea, and the posterior focus at F', about 22.8 mm. behind the cornea.

Note. — Since the investigations of this book are generally for the case where the principal planes and the nodal planes coincide (lens in air), the term *nodals* has been used indiscriminately for the coincident points H and N.



72. Focal Length of Thick Lens.

For parallel incident rays, the image by refraction from the first surface will (§ 12) be at a distance f from the surface, and

$$\frac{\mu - 1}{r} = \frac{\mu}{f}$$

Therefore the distance of the first image from the second surface will be

$$f + \mathrm{e}$$

(If the light comes from the left, this distance will be $f - e$.)

If $v =$ the distance of second image from second surface, then, § 11

$$\frac{\mu}{f + e} - \frac{1}{v} = \frac{\mu - 1}{s} = -\frac{\mu}{f'}$$

Therefore $$\frac{1}{v} = \frac{\mu}{f + \mathrm{e}} + \frac{\mu}{f'} = \mu\frac{\mathrm{e} + f + f'}{f'(f + e)}$$

Whence $$v = \frac{f'(f + \mathrm{e})}{\mu(f + f' + \mathrm{e})} = A'F$$

$f = \dfrac{\mu r}{\mu - 1} =$ focal rad. for 1st surf.

$f' = \dfrac{-\mu s}{\mu - 1} =$ neg. focal rad. of 2d surf.

$\mathrm{e} =$ thickness of lens

$r =$ rad. of curv. of 1st surf.

$s =$ rad. of curv. of 2d surf.

$F =$ principal focus $=$ focus for parallel rays

Therefore

$$N'F = A'F - A'N' = \frac{f'\,(f + \text{e})}{\mu\,(f + f' + \text{e})} - \frac{\text{e}f'}{\mu\,(f + f' + \text{e})}$$

$$= \frac{ff'}{\mu\,(f + f' + \text{e})} = F$$

which is called the focal length, for reasons indicated in § 57.

$\left(\text{If the light comes from the left, we have } F = \dfrac{ff'}{\mu\,(f + f' - \text{e})}\right)$

EXAMPLES

1. Negative lens, $r = 5$, $s = 7$, $\mu = 1.5$, $e = .2$.

Ans. $f = \dfrac{1.5 \times 5}{.5} = 15$, $f' = -\dfrac{1.5 \times 7}{.5} = -21$, $e + f + f' = .2 + 15 - 21 = -5.8$, $A\,N = .34$, $A'N' = .48$, $F = 36.2$. Notice that both nodal points are outside the lens. Light from the right.

2. Negative lens, $r = 5$, $s = -7$, $\mu = 1.5$, $e = .2$.

Ans. Light from right. $f = 15$, $f' = 21$, $A\,N = -.055$, $A'N' = .077$, $F = 5.80$. Notice that the nodal points are both inside the lens and close to the surfaces.

3. Light from right. $r = 7$, $s = 5$, $\mu = 1.5$, $e = .2$.

Ans. Positive lens. $f = 21$, $f' = -15$, $A\,N = -.451$, $A'N' = -.323$, $F = -33.87$. Nodal points are both outside and behind the lens.

4. Light from right. $r = -7$, $s = 5$, $\mu = 1.5$, $e = .2$.

Ans. Double convex lens. $f = -21$, $f' = -15$, $A\,N = -.078$, $A'N' = .056$, $F = -5.85$. Nodal points are inside and very near the surfaces.

5. Light from left. $r = 5$, $s = 7$, $\mu = \frac{3}{2}$, $e = .2$.

Ans. $f = 15$, $f' = -21$, $AN = -.32$, $A'N' = -.45$. Nodal points outside the lens.

6. Light from right. $r = -7$, $s = -5$, $\mu = \frac{3}{2}$, $e = .2$.

Ans. $f = -21$, $f' = 15$, $AN = -.48$, $A'N' = -.34$.

7. Double convex lens. $r = -\frac{3}{4}$, $s = 1$, $e = \frac{1}{2}$, $\mu = \frac{3}{2}$. Light from right.

Ans. $f = -\frac{9}{4}$, $f' = -3$, $AN = -\frac{3}{19}$, $A'N' = \frac{4}{19}$, $F = -\frac{18}{19}$.

8. Double convex lens. $r = \frac{3}{4}$, $s = -1$, $e = \frac{1}{2}$, $\mu = \frac{3}{2}$. Light from left.

Ans. $f = \frac{9}{4}$, $f' = 3$, $AN = \frac{3}{19} = 1.58$, $A'N' = -\frac{4}{19} = -0.21$, $F = 0.947$.

9. Negative lens. Light from left. $r = -\frac{9}{8}$, $s = \infty$, $e = .1$, $\mu = \frac{8}{5}$.

Ans. $f = -3$, $f' = \infty$, $AN = 0$, $A'N' = -\frac{1}{16} = -0.0625$, $F = -\frac{15}{8} = -1.875$.

10. Double convex lens. $r = -\frac{3}{4}$, $s = 1$, $e = \frac{1}{2}$, $\mu = \frac{3}{2}$. Therefore light from right.

Ans. $f = -\frac{9}{4}$, $f' = -3$, $AN = -\frac{3}{19} = -0.157$, $A'N' = \frac{4}{19} = 0.210$, $F = -0.947$.

11. Double convex lens. $r = -\frac{3}{4}$, $s = 10$, $e = \frac{1}{2}$, $\mu = \frac{3}{2}$. Therefore light from right.

Ans. $f = -\frac{9}{4}$, $f' = -30$, $AN = -\frac{3}{127} = -0.0236$, $A'N' = \frac{40}{127} = 0.315$.

12. Double convex lens. $r = -\frac{3}{4}$, $s = 100$, $e = \frac{1}{2}$, $\mu = \frac{3}{2}$. Therefore light from right.

Ans. $f = -\frac{9}{4}$, $f' = -300$, $AN = -0.00248$, $A'N' = .331$.

(Examples 10, 11, 12 are to show how the flattening of the lens causes the node to approach one face.)

13. Plano convex lens. $r = 16$, $s = \infty$, $e = 2$, $\mu = \frac{3}{2}$. Therefore light from left.

Ans. $f = 48$, $f' = \infty$, $AN = 0$, $A'N' = \frac{4}{3}$, $F = 32$.

Notice that in a plano convex the nodes are independent of the finite radius. Ditto, plano concave.

14. Positive meniscus. $r = 10$, $s = 16$, $e = 2$, $\mu = \frac{3}{2}$. Therefore light from left and lens convex towards the left.

Ans. $f = 30$, $f' = -48$, $AN = -2$, $A'N' = -3.2$, $F = 48$. Both nodes outside.

15. Non-curvature lens. $r = 10$, $s = 10$, $e = 2$, $\mu = \frac{3}{2}$. Therefore as in Ex. 14.

Ans. $f = 30$, $f' = -30$, $AN = -20$, $A'N' = -20$, $F = 300$. Therefore as in Ex. 14 both nodes outside.

16. Double convex lens. $r = 10$, $s = -16$, $e = 2$, $\mu = \frac{3}{2}$. Therefore light from left.

Ans. $f = 30$, $f' = -48$, $AN = \frac{10}{19}$, $A'N' = -\frac{16}{19}$, $F = \frac{240}{19}$. Nodes inside.

17. Double convex lens. $r = -10$, $s = 16$, $e = 2$, $\mu = \frac{3}{2}$. Therefore light from left.

Ans. $f = -30$, $f' = -48$, $AN = \frac{1}{2}$, $A'N' = -\frac{4}{5}$, $F = -12$. Nodes inside.

18. Plano convex. $r = \infty$, $s = -16$, $e = 2$, $\mu = \frac{3}{2}$. Therefore light from left.

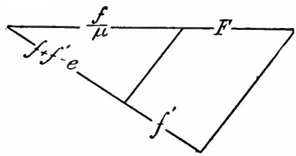
Ans. $f = \infty$, $f' = -48$, $AN = \frac{4}{3}$, $A'N' = 0$, $F = 32$. Nodes inside, one tangent.

19. Concentric lens (Ross lens). $r = 3$, $s = 1$, $e = 2$, $\mu = \frac{3}{2}$.

Ans. $f = 9$, $f' = -3$, $AN = 3$, $A'N' = 1$, $F = -4\frac{1}{2}$. The nodes coincide at the center.

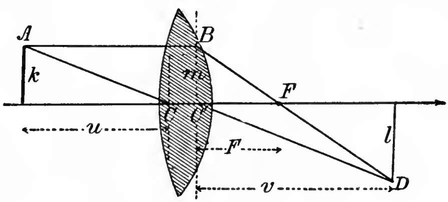
20. In Ex. 8, if $u = 2.594$, whence $v = 1.492$, show that (§ 75) $xy = 0.947^2$.

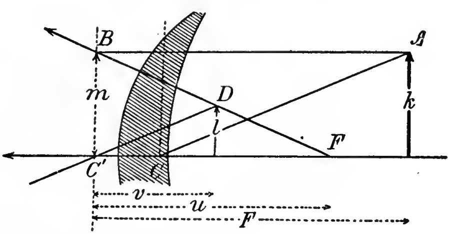
73. Graphic Check. — To detect large errors check the calculation by similar triangles, drawn to scale, in which the sides are as shown.



A large error will be quickly detected in this way before it has time to vitiate the following calculations.

74. Construction for Image (Conf. § 25).





$C'D$ is parallel to AC (§ 63).

By sim. triangles

$$-\frac{u}{v} = \frac{k}{l} = \frac{m}{l} = \frac{F}{v - F} \text{ (upper diag.)}$$

$$\frac{u}{v} = \frac{k}{l} = \frac{m}{l} = \frac{F}{F - v} \text{ (lower diag.)}$$

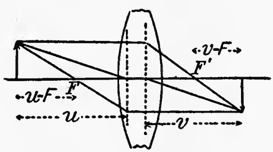
Whence

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

Hence *the distances of object and image from the nodal points obey the same law as the distances from the lens in the case of thin lenses,* the focal length being the distance from the nodal point of emergence to the principal focus.

The nodal planes take the place of the two coincident faces of the thin lens, and the constructions and calculations are carried on as if the thin lens were split and then the two edges of the split separated the distance between the nodal planes.

75. Exercise. — From the diagram show that



$$(\mu - F)\,(v - F) = F^2$$

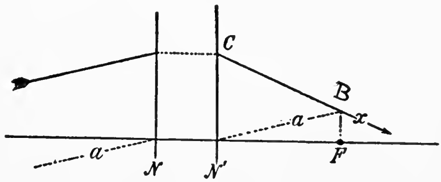
or, as it is generally written, $xy = F^2$, x and y being the distances of the object and image from the focal points.

76. Use of Formulae. — Decide upon the direction of light and give the corresponding signs to f, f' (see §§ 36, 37). Then select the proper formulae corresponding to the direction of the light. (Light from the left makes the f of the positive lens $+$, a seeming gain in concordance of signs.)

77. Graphic Tracing of any Ray Path.

This follows the formula of § 32, $rl \rightarrow \phi_2 \,||$, exactly, except that the lens line is split apart the distance between the nodal planes, the points in one nodal plane being

dragged horizontally to the other. The order of the letters indicates the order of construction, x being the line sought.



78. Since the nodal planes are really plane surfaces, their intersections with the paper will be straight lines, as drawn in the diagrams. Therefore, having the nodals and foci of two lenses given in position, we can find the nodals of the combination by § 61, by taking the initial rays horizontal.

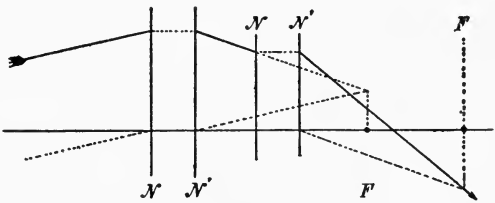


Diagram showing the tracing of a ray through two successive lenses, the principal planes of each lens being indicated by letters in horizontal lines.

Example 1. — Try this on the combinations of § 107, Exs. 4, 8.

Example 2. — See § 95, Ex. 2.

These two sections are an extension of the principles of § 32, and are equally important in the application to nodal planes.

ANALYTICAL INVESTIGATION[1]

79. The previous investigation has assumed some facts

[1] The remaining sections of this chapter are for those inquisitive readers who desire a somewhat more rigorous logic and less depend-

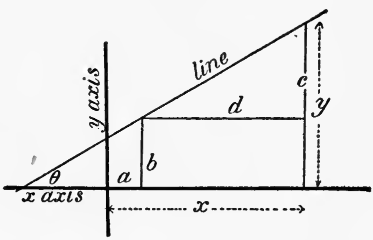
as self-evident. The investigation of this section is for the purpose of putting these facts on a more strictly logical basis, to meet the criticism to which the preceding sections might be open to the casuist.

80. Before entering upon the discussion, we give some preliminary principles.

y = dist. above (or below) the x axis, of an arbitrary point on the line
x = distance of the point, to right or left of the y axis
a, b = corresponding distances for some fixed point
x and y have many values, one for each point
a and b are constant, fixing some definite point

By sim. triangles $$\frac{y - b}{x - a} = \frac{c}{d} = \tan \theta = m$$

or, as it is generally written $$y - b = m\,(x - a)$$



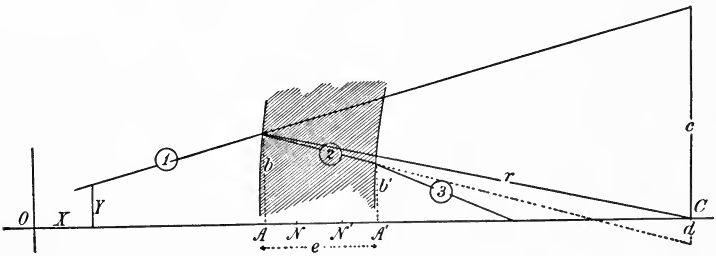
This is called the *equation of the line* referred to the axis, since x and y taken in corresponding values fix any point on the line. Their values could be used to plot points on the line; or corresponding values measured from a point on the line will satisfy the equation.

From the equation we can locate the line by assuming a value for x and calculating the corresponding value of y, and then plotting the two values, thus locating a point, and so on. In other words, an equation of a line gives us a clue as to where the line lies.

ence upon intuition. They can safely be omitted by those not interested, without destroying the continuity of the text.

The advantage of the equation is that we can operate upon the equation algebraically and then interpret the result geometrically, without going through all the peculiarities of a geometrical diagram.

81. General Equation of a Refracted Ray.



①, ②, and ③ represent the ray before, during, and after refraction

For rays through points *near* the vertex A, so that the point of incidence is practically over A,

1. The equation to line ① is (§ 80):

$$y - b = m\,(x - OA)$$

2. For line ②

$$y - b = m'\,(x - OA) \text{ or } (y - b') = m'\,(x - OA')$$

3. For line ③ $\quad y - b' = m''\,(x - OA')$

4. From equation 2, $\quad b - b' = m'\,(OA' - OA) = m'e$

By § 4 $\qquad \sin r① = \mu \sin r②$

[$r①$ means the angle between r and ①, etc.

But by § 6 $\qquad \dfrac{c}{r} = \dfrac{\sin r①}{\sin c①}$

Therefore $\dfrac{c}{r}\sin c① = \sin r① = \mu \sin r②$

5. $\qquad = \mu \dfrac{d}{r} \sin d② \qquad \left[\dfrac{d}{r} = \dfrac{\sin r②}{\sin d②},\right.$ § 6

Therefore $c = b + m\,(OC - OA)$ [Eq. 1 taking $y = c$

$= b + mr$

Similarly $d = b + m'r$

Therefore $(b + mr) \sin c① = \mu\,(b + m'r) \sin d②$

or $b + mr = \mu\,(b + m'r)$ [Since $\sin c① = \sin d②$, practically, the two angles being nearly 90° each

6. Whence $\mu m' = m - \dfrac{\mu - 1}{r} b = m - bu$

[Making $\dfrac{\mu - 1}{r} = u$

7. Similarly $\mu_1 m' = m'' - b'u'$

[$\mu' = \dfrac{\mu_1 - 1}{s}$, $\mu_1 =$ index of refraction for 2d surface

$s =$ radius of 2d surface

Now $b' = b + \dfrac{m - bu}{\mu} e = b\left(1 - \dfrac{eu}{\mu}\right) + \dfrac{me}{\mu}$ [Eqs. 4, 6

8. $= gb + hm$ [Putting $1 - \dfrac{eu}{\mu} = g, \dfrac{e}{\mu} = h$

$m'' = \mu_1 m' + bu' \dfrac{(1 - eu)}{\mu} + \dfrac{meu'}{\mu}$ [Eqs. 8, 7

$= \mu_1 \dfrac{m - bu}{\mu} + bu'\left(1 - \dfrac{eu}{\mu}\right) + \dfrac{meu'}{\mu}$

9. $= m\,\dfrac{\mu_1 + eu'}{\mu} + b\left(u' - u\dfrac{\mu_1}{\mu} - \dfrac{euu'}{\mu}\right)$

10. $= kb + lm$

[Putting $k = u' - \dfrac{u\mu_1}{\mu} - \dfrac{euu'}{\mu} = u'g - u\dfrac{\mu_1}{\mu}$

$l = \dfrac{\mu_1 + eu'}{\mu}$

If $b = Y - m(X - OA)$ $\left[\begin{array}{l} X, Y \text{ being co-ordinates of the} \\ \text{point on the line which is} \\ \text{considered as the source of} \\ \text{the ray} \end{array}\right.$

then
$$b' = gY + m(h - g \cdot \overline{X - OA})$$
$$m'' = kY + m(l - k \cdot \overline{X - OA})$$

From these
$$m = \frac{m'' - kY}{l - k(X - OA)}$$
$$b' = gY + (m'' - kY)\frac{h - g(X - OA)}{l - k(X - OA)}$$

Whence
$$y - Y\left(g - k\frac{h - g(X - OA)}{l - k(X - OA)}\right) =$$
$$m''\left(x - OA' + \frac{h - g(X - OA)}{l - k(X - OA)}\right) \qquad \text{[Eq. 3}$$

11. Or
$$y - \frac{Y\mu_1}{\mu[l - k(X - OA)]} = m''\left(x - OA' + \frac{h - g(X - OA)}{l - k(X - OA)}\right)$$

$\left[\text{Since } gl - hk = \frac{\mu_1}{\mu}\right.$

the equation of the emerged ray in terms of X, Y, the co-ordinates of the source.

82. If X be taken such that $l - k(X - OA) = \frac{\mu_1}{\mu}$

that is
$$X = OA + \frac{l - \frac{\mu_1}{\mu}}{k} = OH, \text{ say}$$

then when
$$x = OA' - \mu\frac{h - g(X - OA)}{\mu_1}$$

$$= OA' - \frac{\mu}{\mu_1}\left(h - g\,\frac{l - \frac{\mu_1}{\mu}}{k}\right) \qquad \left[X - OA = \frac{l - \frac{\mu}{\mu_1}}{k}\right.$$

$$= OA' - \frac{\mu}{\mu_1}\left(\frac{hk - gl + g\frac{\mu_1}{\mu}}{k}\right)$$

$$= OA' - \frac{\mu}{\mu_1}\left(\frac{-\frac{\mu_1}{\mu} + g\frac{\mu_1}{\mu}}{k}\right) \qquad \left[\text{Since } gl - hk = \frac{\mu_1}{\mu}\right.$$

$$= OA' + \frac{1-g}{k} = OH', \text{ say}$$

we will have $$y = Y$$

and in the planes of these two points (H and H') the object (Y) and the image (y) are equally distant from the axis of the lens; the rays are transferred horizontally. (Conf. §§ 62, 69.)

These points are called *principal points*, and perpendicular planes through them *principal planes*.

83. If X be taken such that $l - k\,(X - OA) = 1$,

whence $$X = OA + \frac{l-1}{k} = ON, \text{ say}$$

then when

$$x = OA' - \left\{h - g\,(X - OA)\right\} = OA' - \left(h - g\,\frac{l-1}{k}\right)$$

$$\left[X - OA = \frac{l-1}{k}\right.$$

$$= OA' - \frac{hk - gl + g}{k} = OA' + \frac{\frac{\mu_1}{\mu} - g}{k}$$

$$= ON', \text{ say} \qquad \left[gl - hk = \frac{\mu_1}{\mu}\right.$$

and when $Y = 0$, then also $y = 0$, and

$$m = \frac{m'' - kY}{l - k(X - OA)} = m''$$

since $\qquad kY = 0$ and $l - k(X - OA) = 1$

84. Since $m = m''$, the rays before refraction and after refraction are parallel (Conf. § 63), and the image is not deflected so long as this point N' is not moved. (Conf. § 59.)

The points N and N' are called the *nodal points.*

85. If $\mu_1 = \mu$ (e.g. air on both sides), then H and H' coincide respectively with N and N'. (Conf. §§ 64, 71.)

86. If $m'' = 0$, i.e. the ray is parallel to the axis after refraction, then from eq. 10

$$b = -\frac{l}{k}m$$

and the equation of the incident ray is

$$y + \frac{lm}{k} = m(x - OA) \quad \text{or} \quad y = m\left(x - OA - \frac{l}{k}\right)$$

87. If we also take $y = 0$, so as to find where the ray crosses the axis, then

$$x = OA + \frac{l}{k} = OA + \frac{\frac{\mu_1}{\mu} + \frac{eu'}{\mu}}{u' - u\frac{\mu_1}{\mu} - \frac{euu'}{\mu}} \qquad \text{[Eq. 10}$$

$$= OF, \text{ say}$$

88. If $m = 0$, i.e. the incident ray parallel to the axis, then from eqs. 8, 10

$$b' = gb = \frac{gm''}{k}$$

and the equation of the refracted ray becomes, eq. 3

$$y - \frac{gm''}{k} = m''(x - OA') \text{ or } y = m''\left(x - OA' + \frac{g}{k}\right)$$

whence if also $y = 0$, in order to find where the ray crosses the axis, then the distance to the crossing point is

$$x = OA' - \frac{g}{k} = OA' - \frac{\frac{\mu_1}{\mu} - \frac{eu}{\mu}}{u' - u\frac{\mu_1}{\mu} - \frac{euu'}{\mu}} = OF', \text{ say.}$$

F and F' are called the *focal points.*

89. For $\mu_1 = \mu$, the usual case

$$OF = OA + \frac{\mu + eu'}{\mu(u' - u) - euu'} =$$

$$OA - \frac{f(f' - e)}{\mu(f + f' - e)}$$

$$OF' = OA' - \frac{\mu - eu}{\mu(u' - u) - euu'} =$$

$$OA' + \frac{f'(f - e)}{\mu(f + f' - e)}$$

$$OF' - ON' = OA' - \frac{g}{k} - \left(OA' + \frac{1 - g}{k}\right) = ON - OF$$

$$= \frac{-1}{k}$$

$$= \frac{-\mu}{\mu(u' - u) - euu'} = \frac{ff'}{\mu(f + f' - e)} = F$$

$=$ equivalent focal length of the lens

(Conf. § 72)

90. From eq. 11

$$y = m''\left\{x - \left(OA' - \frac{h - g\,(X - OA)}{l - k\,(X - OA)}\right)\right\} + \frac{Y}{l - k\,(X - OA)}$$

$$= m''\,(x - \xi) + \eta \qquad \left[\begin{aligned} \xi &= OA' - \frac{h - g\,(X - OA)}{l - k\,(X - OA)} \\ \eta &= \frac{Y}{l - k\,(X - OA)} \end{aligned}\right.$$

where ξ, η are evidently (since $x = \xi$, $y = \eta$ satisfies the equation) on the ray ③, and evidently dependent only on X and Y and not on m, b; that is, every ray through X, Y (the object) passes through ξ, η (the image).

From $OA = ON - \frac{l - 1}{k}$, $OA' = ON' - \frac{1 - g}{k}$

substituting these values in the expressions for ξ, η, we get

$$\xi = ON' - \frac{ON - X}{1 + k\,(ON - X)} \qquad \eta = \frac{Y}{1 + k\,(ON - X)}$$

whence $$\frac{1}{\xi - ON'} - \frac{1}{X - ON} = -\,k$$

or $$\frac{1}{p} - \frac{1}{p_1} = -\,k = \frac{1}{F}$$

where p = distance from node N' to the image ξ, and p_1 = distance from the object X to the node N. This shows that *the nodal distances to object and image obey the same laws as the thin lens distances.* (Conf. § 74.)

CHAPTER IV

COMBINATIONS OF LENSES

91. Thin Lenses in Contact.

For the first lens $$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

for the second lens $$\frac{1}{v_2} - \frac{1}{v_1} = \frac{1}{f_2}$$

whence, by addition $$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{F}$$

f_1, f_2 = focal lengths of the lenses
u = dist. of object from 1st lens
v_1 = dist. of 1st image from 1st lens and of 2d object from 2d lens
v_2 = dist. to image formed by 2d lens
F = focal length of combination

Example. — A + lens, 2 in. focus, is cemented to a − lens, 9 in. focus. What is the equivalent focus?

Ans. Equivalent focus = $2\frac{4}{7}$.

92. For a third lens, similarly

$$\frac{1}{v_3} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{F}$$

The power of the combination is the sum of the powers of the components. (Conf. § 42.)

For powers of lenses not in contact, see § 96.

93. Thin Lenses not in Contact.

Taking A, a point on the refracted ray and in the front focal plane of the second lens,[1] we can find its image, C, through the second lens by known rays, as shown (or by § 32). But any other ray through A, as AB, must go through the same image point, and thus we get the direction BC for this AB ray after it has been refracted by the second

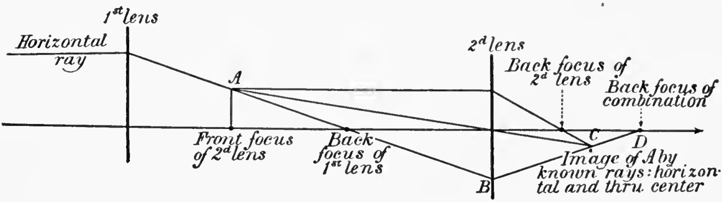


Diagram illustrating general case of two thin lenses, μ not equal to μ_1; hence front and back focal length of second lens will have different values. Introduced for the purpose of getting a general rule of construction for use in the next diagram, indicated by the order of the letters.

lens. The point D, where it meets the axis, is the *back focus* of the combination for incident parallel rays.

EXAMPLES

1. A candle is held 1 foot in front of a convex lens, giving an image on a screen 4 inches behind it. A concave lens is now placed in contact with it, and the screen must be moved 8 inches further away to get the image. What is the focal length of the negative lens? Check by § 32. *Ans.* -6.

2. A convex lens of 16 cm. focus, in contact with a negative lens, gave a focal length of 48 cm. for the

[1] In order to have its distance from the lens a definite and significant value.

combination. What is the focal length of the negative lens? *Ans.* -24.

3. A concave lens of -8 cm. is combined with a convex lens of $+6$. What is the focus of the combination? *Ans.* 24.

4. By the method of §§ 32, 33, show that a compound microscope (two positive lenses more than the sum of their focal distances apart) must have the object without the focal distance of the objective. (Conf. § 33, Ex. 2.)

5. When the distance outside the focus in Ex. 4 becomes infinity, the distance apart of the lenses becomes the sum of the focal radii, and we have the celestial telescope.

6. Show by §§ 32, 33 that a positive lens followed by a negative lens, the distance apart being the difference of the foci or less, will give a virtual image. (Ordinary opera glass.)

7. If the distance apart in Ex. 6 is greater than the difference, there results a real image. (Telephoto, §§ 113, 29.)

8. In the Huygens eyepiece (field lens focus $= 3f$, eye lens focus $= f$, distance between lenses $= 2f$), show by § 32 that rays incident upon the front lens pointed toward a point between the lenses $\frac{3}{2}f$ from the front lens will emerge from the second lens parallel to the axis; that is, will give a virtual image.

9. In the Ramsden eyepiece (field lens focus $=$ eye lens focus $= f$, distance apart of lenses $= \frac{2}{3}f$), show by § 32 that parallel rays incident upon the front lens converge to a focus $f/4$ beyond the back lens.

10. By §§ 32, 25, etc., show that the second lens of Ex. 6 reinverts the image made by the $+$ lens, thus giving a final erect (virtual) image.

11. Similarly show that in Ex. 7 the real image is kept inverted.

94. Back Focal Distance for Two Thin Lenses. For construction of diagram, see § 93.

By similar triangles $$\frac{f_1 - \epsilon}{f_1 + f_2 - \epsilon} = \frac{d}{e} = \frac{d}{d+g} = \frac{\mathfrak{B}}{f_2}$$

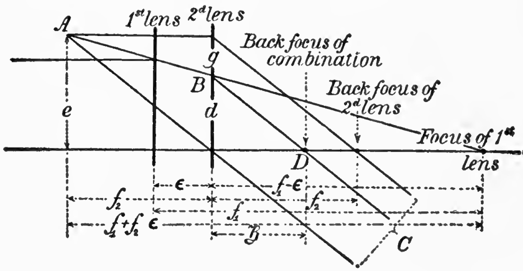
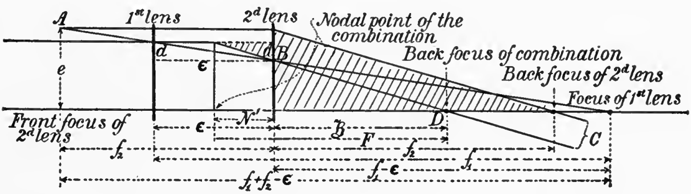


Diagram showing the construction for back focal distance when $\mu_1 = \mu$. The letters A, B, C designate the same points as in the preceding diagram.

Therefore $\mathfrak{B}$ = back focal distance of combination = distance from lens to focal point

$$= \frac{f_2\,(f_1 - \epsilon)}{f_1 + f_2 - \epsilon}$$

95. Equivalent Focus for Two Thin Lenses. — The two



thin lenses act like the two surfaces of a thick lens, and, like them, have their corresponding nodal points determined in the same way (see § 61), remembering that the

surfaces are now vertical plane surfaces. Add the construction of § 61 to the diagram of § 94.

For construction of diagram, Conf. §§ 93, 94, 61.

By similar triangles (lightly shaded in the diagram for the second set)

$$\frac{\epsilon}{f_1 + f_2 - \epsilon} = \frac{d}{e} = \frac{\mathrm{N}'}{f_2}$$

Therefore $\quad \mathrm{N}' = \dfrac{f_2\epsilon}{f_1 + f_2 - \epsilon}$ = nodal distance from corresponding lens

Therefore $\quad F = \mathfrak{B} + N' = \dfrac{f_2(f_1 - \epsilon)}{f_1 + f_2 - \epsilon} + \dfrac{f_2\epsilon}{f_1 + f_2 - \epsilon} =$

$\dfrac{f_1 f_2}{f_1 + f_2 - \epsilon}$ = equivalent focal length of the combination

= distance from nodal point to focal point.

If we take into account the direction, we get

$$N' = \frac{-f_2\epsilon}{f_1 + f_2 - \epsilon}$$

and similarly

$$N = \frac{f_1\epsilon}{f_1 + f_2 - \epsilon}$$

This section, with § 61, enables us to find the nodal points of a combination of lenses.

Ex. 1. In the Huygenian eyepiece (field lens focus $= 3f$, eye lens focus $= 2f$, $\epsilon = 2f$), show that the front focal distance (found by parallel rays through the eye lens, from right) is $-\frac{3}{2}f$. (Conf. § 93, Ex. 8.)

2. In the Ramsden eyepiece ($f_1 = f_2$, $\epsilon = \frac{2}{3}f_1$), show that the back focal length is $\frac{1}{4}f_1$. That is, the combination has the properties of an ordinary convex lens of $f/4$ focus. (It has the advantage, however, of being approximately achromatic.) (Conf. § 93, Ex. 9.)

3. By the method of § 61, show that if two thin positive lenses lie with crossed foci, and each lens within the focus of the other, the resulting foci of the combination will lie outside the lenses, and both nodal points between the lenses.

4. In the preceding example, if each lens is without the focus of the other, then both foci of the combination are between the lenses, and both nodals are outside, and the nodals are crossed.

5. By §§ 32, 25, etc., show that a microscope composed of a $\frac{1}{2}$ in. objective and a 1 in. eyepiece, 6 inches apart, for a person of 8 in. vision must have the object $\frac{46}{83}$ in front of the objective.

EXAMPLES

1. Two positive lenses with a common focal length of 0.05 m. are 0.05 m. apart. What image results of a disc 0.01 m. in diameter placed 0.1 m. distant?

Ans. A real image 0.025 m. beyond the second lens. Diameter of image = 0.005 m.

Note the crossing of the nodes. Draw a diagram and compute the magnification by similar triangles. Conf. diagram of § 74, *mutatis mutandis.*

2. (Microscope ocular) Field lens, $f_1 = 2\frac{1}{4}$, eye lens, $f_2 = 1\frac{3}{8}$, $\epsilon = 2\frac{1}{8}$. Show graphically (see §§ 95, 33, 77) that the nodes cross, N' to just behind (right of) the field lens, N to about an inch behind (right of) the eye lens (light from left); that the posterior focus almost coincides with the focus of the field lens; the anterior focus not so closely with the anterior focus of the eye lens. Calculate these results by the formulae of § 95.

96. Powers of Thin Lenses not in Contact.

From § 95

$$\frac{1}{F} = \frac{f_1 + f_2 - \epsilon}{f_1 f_2} = \frac{1}{f_2} + \frac{1}{f_1} - \frac{\epsilon}{f_1 f_2} = p_1 + p_2 - p_1 p_2 \epsilon$$

= power of the combination

$\left[\begin{array}{l} p_1,\ p_2 = \text{powers of the thin lenses} \\ \quad\ \ \epsilon = \text{distance apart} \end{array}\right.$

Example 1.—$p_1 = 3\ D$, $p_2 = 5\ D$, $\epsilon = .025$ m.

$\therefore$ Equiv. power $= 3 + 5 - 15 \times 0.025 =$ 7.625 diopters

Example 2.—$p_1 = p_2 = +12\ D$, $\epsilon = .02$ m.

$\therefore$ Equiv. power $= 12 + 12 - (12 \times 12 \times .02) =$ 21.12 diopters

97. Back Focal Distance for Light from the Right.

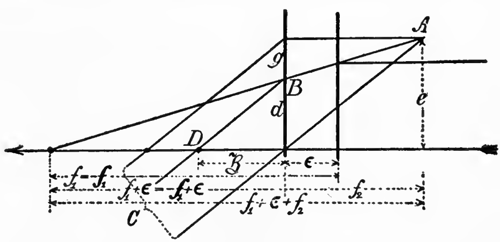


Diagram constructed exactly as in § 94, *mutatis mutandis.*

By sim. triangles $\qquad \dfrac{f_1 + \epsilon}{f_1 + f_2 + \epsilon} = \dfrac{d}{e} = \dfrac{d}{d + g} = \dfrac{\mathfrak{B}}{f_2}$

Whence $\qquad \mathfrak{B} = \dfrac{f_2\,(f_1 + \epsilon)}{f_1 + f_2 + \epsilon}$

98. Equivalent Focus for Light from the Right.

Similarly to § 95 $\qquad F = \dfrac{f_1 f_2}{f_1 + f_2 + \epsilon}$

Note the change of sign of ϵ in the formulae when the light comes from the right, due to the fact that ϵ is normally positive.

99. Back Focal Distance for Two Thick Lenses.

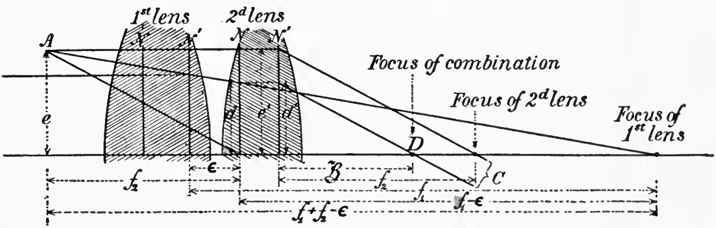


Diagram showing construction for back focus of thick lens, on the same principle as the preceding diagrams; but note the horizontal transference between the nodal planes in the second lens. The nodal planes are designated by N, N'. For construction see § 94.

By similar triangles $$\frac{f_1 - \epsilon}{f_1 + f_2 - \epsilon} = \frac{d}{e} = \frac{d'}{e'} = \frac{\mathfrak{B}}{f_2}$$

whence $\mathfrak{B}$ = back focal distance =

$$\frac{f_2\,(f_1 - \epsilon)}{f_1 + f_2 - \epsilon} \quad \text{(with light from the left)}$$

= distance from posterior nodal point of *lens* to focal point

100. Equivalent Focal Length for Two Thick Lenses. Similarly, as in § 95

$$F = \frac{f_1 f_2}{f_1 + f_2 - \epsilon} \quad \text{(with light from the left)}$$

= distance from posterior nodal point of the *system* to the focal point

remembering that F is measured from the nodal plane of emergence for the combination, just as f_2 and f_1 were

measured from the nodal planes of emergence of the corresponding lenses.

Hence $F = \dfrac{f_1 f_2}{f_1 + f_2 + \epsilon}$ (with light from the right)

101. Nodal Distances for Thick Lenses.

As in § 95, $N'\mathfrak{N}'$ = distance between N' of second lens and nodal point of emission of the *combination*, taking account of direction

$$= \frac{-\epsilon f_2}{f_1 + f_2 - \epsilon}$$

$N\mathfrak{N}$ = distance from N of first lens to nodal point of incidence of the *combination*

$$= \frac{\epsilon f_1}{f_1 + f_2 - \epsilon}$$

102. Graphic Check. — Check for large errors, by similar triangles as in §§ 73, 68, except that there is no μ in the formula.

103. Equivalent Thickness of Thick-Lens Combination.

$\mathfrak{N}\mathfrak{N}'$ = distance between the nodal points of the combination

$$= \epsilon + N N' \text{ (1st lens)} + N N' \text{ (2d lens)} - \frac{\epsilon f_1}{d} - \frac{\epsilon f_2}{d}$$

$[d = f_1 + f_2 - \epsilon$

$$= N N' \text{ (1st lens)} + N N' \text{ (2d lens)} - \frac{\epsilon^2}{d}$$

$$= N N' \text{ (1st lens)} + N N' \text{ (2d lens)} - \frac{\epsilon^2 F}{f_1 f_2}$$

$$\left[\begin{array}{ll} F & = \text{focal length of combination} \\ f_1, f_2 & = \text{focal lengths of components} \\ \epsilon & = \text{distance between the } N' \text{ of the one} \\ & \quad \text{component and the } N \text{ of the 2d} \\ & \quad \text{component} \end{array}\right.$$

This value should be used as a check in the computation to compare with the value derived from the diagram by introducing the various values, $A\,N$, etc. It must be used, of course, in connection with the antecedent check of § 68.

Example 1.—$f_1 = 4, f_2 = 3,\ N\,N'$ (1st lens) $= .15$, 2d lens $= .2,\ \epsilon = 1.5$.

Ans. $F = 2.18,\ \mathfrak{N}\,\mathfrak{N}' = -\ 0.059$.

104. Power of Thick-Lens Combination.

From § 100 $\quad P = \dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{\epsilon}{f_1 f_2} = p_1 + p_2 - p_1 p_2 \epsilon$

If both lenses are +, increase of ϵ increases the equivalent focal length and reduces the equivalent power.

If one lens is −, so that the term $-\ p_1 p_2 \epsilon$ becomes positive, increase of ϵ will shorten the equivalent focal length and increase the power.

The value of $\mathfrak{N}\,\mathfrak{N}'$ (§ 101) shows that the equivalent thickness of a combination of two + lenses is reduced by separating them, and may become zero or negative if ϵ is large enough; i.e. the two nodal planes will cross each other, as is the case in many camera lenses (see Ex. 7, § 106), microscope oculars, etc.

RÉSUMÉ

105. Notice that the thick lens acts like a thin lens with its surfaces (plane and perpendicular to the axis, because we are considering only points near the axis) split apart and separated the distance between the nodal (see § 64)

points, so that if we should make the construction for a thin lens and then split the diagram along the lens line and pull it apart the distance between the nodes, we would have the appropriate diagram for the thick lens. We call this the *equivalent thin-split.* The nodal distance can, of course, only be found by means of the equations of § 67, after **e** has been decided upon.

A combination of two lenses again acts like two thin lenses (with separated faces, the nodal distance) removed from each other the distance (ϵ, see § 95) between the posterior nodal point of the first lens and the anterior nodal point of the second lens (see diagram of § 99), this being the distance between the inner faces of the thin lenses before the faces were split apart to act as a thin-split. This distance, ϵ, can be assumed as we please, and then from the equations of § 101 we can calculate the nodal points which act as the front and rear face of the new equivalent thin-split lens. And so on.

That is, we determine the equivalent thin-split lens of the individual lenses, and then the equivalent thin-split lens of the new combination, and so on, the final thin-split lens being the equivalent of the combination.

Notice that in the equations of §§ 94–100, μ has disappeared because the medium between the two thin-split lenses is air (for which $\mu = 1$) or its equivalent, even if the lenses be in contact.

106. Use of Formulae for Combined Lenses.

(a) Find f, f', F, $\overline{A\,N}$, $\overline{A'\,N'}$ for each lens. (Conf. § 76.)

f, § 12
f', § 15
F, §§ 16, 72
$\overline{A\,N}$, $A'\,N'$, § 67

f, f' = focal radii for surface refraction.

e = thickness of component lens.

F = focal length of the lens (or later of the combination, or thin-split lens). (§§ 16, 72, 100.)

A, A' = anterior and posterior vertex of lens (or combination of lenses).

$\overline{AN}$, $\overline{A'N'}$ = nodal distances from anterior and posterior vertex of lens (or later of the combination). (§§ 67, 101.)

(β) Select f_1, f_2 for the combination, then find (decide upon) ϵ, and then compute F, $\mathfrak{N}$, $\mathfrak{N}'$ for the combination. (§§ 100, 101.)

$\mathfrak{N}, \mathfrak{N}'$ = nodal distances from anterior (of the first lens) and posterior (of the second lens) nodal points of the components to the corresponding nodal points of the combination.

f_1, f_2 = (used in the formula for the combination) the F's of the components.

ϵ = distance between the N' of the anterior component and the N of the posterior component. ϵ must be taken negative when the nodals cross. To calculate ϵ locate the nodals roughly on a diagram, with the distances noted thereon, and then derive the value of ϵ.

(γ) Then find $\overline{AN} + \mathfrak{N}$ = distance of anterior nodal point of combination from the anterior vertex, $\overline{A'N'} + \mathfrak{N}'$ = distance of posterior nodal point of combination from the posterior vertex of the combination. $AN + \mathfrak{N}$ of component = AN of the (next) combination, $A'N' + \mathfrak{N}'$ of the component = $A'N'$ of the combination. (See Exs. 7, 8.)

Do not fail to use the checks of §§ 14, 68, 73, 102, 101, 107.

Repeat this for the combination of this combination with some other, and so on.

Great care must be taken in regard to + and − signs. Here most of the errors occur.

107. Graphic Construction (only available when there is not too great disparity between the numerical values used) after the foci and nodals of the component lenses have been located (based upon the calculations of § 16 for thin lenses, and §§ 67, 72 for thick lenses, because owing to the disparity of the values used the intersections in the graphic work are too acute to be of service).

Graphic construction is extremely valuable to check up which side of the nodals (which limit ϵ) the new nodals of the combination lie. Does not need to be to scale so long as the relation of greater and less is preserved.

In the Case of Thin–Lens Components

Find the equivalent focus of the combination by starting with a horizontal ray, its first refraction being found by §§ 25–30. Where the refracted ray strikes the second lens, find its new course by § 32. The intersection of this new course with the axis will be the focus of the combination.

Find the nodals (principal points) by the principle of § 61 (illustrated in the diagram of § 95).

In the Case of Thick–Lens Components

Find the equivalent focus of the combination by starting with a horizontal ray, its first refraction being found by § 74. Where the refracted ray strikes the nodal line (incident, look out for crossed nodals) of the second lens, find its new course by § 77. The intersection of this new course with the axis will be the focus of the combination.

Find the nodals of the combination by the principle of § 61, prolonging the last ray (second refraction) back to its intersection with the original horizontal ray.

EXAMPLES

1. Two thin positive lenses, 2 inches apart, of focal lengths 6 and 9.

$$A N + \mathfrak{N} = 0 + \frac{2 \cdot 6}{6 + 9 - 2} = \frac{12}{13}$$

$$A' N' + \mathfrak{N}' = 0 + \frac{-2 \cdot 9}{6 + 9 - 2} = -\frac{18}{13}$$

$$F = \frac{6 \cdot 9}{6 + 9 - 2} = \frac{56}{13}$$

The nodals are between the lenses.

2. Thin lenses, 5 inches apart, a positive of 6 focal length and a negative of -2.

$$A N + \mathfrak{N} = 0 + \frac{5 \cdot 6}{6 - 2 - 5} = \frac{+30}{-1}$$

$$A' N' + \mathfrak{N}' = 0 + \frac{-5(-2)}{6 - 2 - 5} = -10$$

$$F = \frac{6(-2)}{-1} = 12$$

Notice that the nodals are outside the combination and far in front, and that the back focal distance is only 2. Distance between nodals = 25.

3. Same as in Ex. 2, but the combination reversed.

$$A N + \mathfrak{N} = 10 \qquad A' N' + \mathfrak{N}' = 30$$

Back focal distance = 42 $\qquad F = 12$

Distance between nodals = 25

4. Two thick lenses.

First lens. $r = 6,\ s = -4,\ e = 3,\ \mu = 1.5$

$$f = 18,\ f' = 12,\ A N = \tfrac{4}{3},\ A' N' = -\tfrac{8}{9} = -.889$$

$$F = \tfrac{16}{3} = 5.33,\ N N' = \tfrac{7}{9}$$

Second lens. $r = -4,\ s = -2,\ e = 1,\ \mu = \tfrac{3}{5}$

$$f = -\tfrac{32}{3},\ f' = \tfrac{16}{3},\ AN = \tfrac{20}{19} = 1.05, \quad A'N' = \tfrac{10}{19} = .525$$

$$F = \tfrac{320}{57} = 5.614,\ NN' = \tfrac{9}{19}$$

Combination. Assume $\epsilon = 2$, thus making the lenses 0.062 apart.

$$f_1 = 5.33,\ f_2 = 5.614$$

$$F = \frac{5.33 \times 5.614}{5.33 + 5.614 - 2} = \frac{29.939}{8.947} = 3.35$$

$$AN + \mathfrak{N} = 1.33 + \frac{2 \times 5.33}{8.947} = 1.33 + 1.19 = 2.52$$

(located in the first lens)

$$A'N' + \mathfrak{N}' = .525 + \frac{-2 \times 5.614}{8.947} = .525 - 1.25 = -.725$$

(in the second lens)

$$\mathfrak{N}\mathfrak{N}' = 0.804$$

5. Two thin lenses, 4 inches apart, a positive of 6 focal length, and a negative of -6 focal length, the positive lens in front.

$$AN = -6,\ A'N' = -6,\ F = 9$$

The nodals are outside the lenses, 6 and 2 respectively, from the front face of the positive lens. $\mathfrak{N}\mathfrak{N}' = 4$.

6. Lens of Ex. 8, § 72, 0.25 in front of lens of Ex. 9. Whence

$$f_1 = 0.947,\ f_2 = -1.875,\ \epsilon = 0.21 + 0.25 + 0 = 0.46$$

Therefore $$F = \frac{.947 \times (-1.875)}{.947 - 1.875 - .46} = \frac{}{-1.388} = 1.28$$

$$AN + \mathfrak{N} = .158 + \frac{.46 \times .947}{.947 - 1.875 - .46} =$$

$$.158 - .314 = -0.156$$

(in front of first lens)

$$A'N' + \mathfrak{N}' = -0.0625 + \frac{-.46 \times (-1.875)}{-1.388} =$$

$$-0.0625 - 0.621 = -0.6835$$

(in first lens)

7. Three lenses, in contact. Light from left.

First lens. $r = -4.29$, $s = -1.20$, $\mu = 1.5146$, $e = .230$ whence $f = -12.627, f' = 3.532, F = 3.157, AN = .2056$,

$A'N' = 0.0573$.

Second lens. $r = -1.20$, $s = -3.75$, $\mu = 1.574$,

$e = 0.050$

Whence $f = -3.2906$, $f' = 10.2831$, $F = -3.094$,

$AN = -0.015$, $A'N' = -0.047$.

Third lens. $r = -3.75$, $s = -1.8$, $\mu = 1.517$, $e = 0.151$

Whence $f = -11.0033$, $f' = 5.2814$, $F = 6.528$,

$AN = 0.186$, $A'N' = 0.089$

Combination of 1st and 2d lens. $f_1 = 3.157$, $f_2 = -3.094$,

$\epsilon = -0.0723$

Whence $F = \dfrac{3.157\,(-3.094)}{3.157 - 3.094 + 0.0723} = \dfrac{}{0.1353} = -71.76$

$$AN + \mathfrak{N} = 0.2056 + \frac{-0.0723 \times 3.157}{0.1353} =$$

$$0.2056 - 1.68 = -1.4744$$

$$A'N' + \mathfrak{N}' = -0.047 - 1.65 = -1.697$$

Notice that ϵ is negative because the N' of the first lens and the N of the second lens are crossed.

First combination, combined with 3d lens.

$f_1 = -71.76$, $f_2 = 6.528$, $\epsilon = 1.697 + 0.186 = 1.883$

Whence $$F = \frac{-71.76 \times 6.528}{-71.76 + 6.528 - 1.883}$$

$$\frac{-\ 468.4492}{-\ 67.115} = 6.97$$

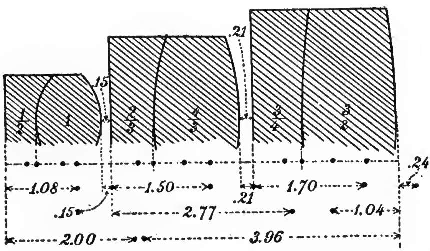
$$A\,N + \mathfrak{N} = -\ 1.474 + \frac{1.883\ (-\ 71.76)}{-\ 67.115} =$$

$$-\ 1.474 + 2.013 = 0.539$$

$$A'\,N' + \mathfrak{N}' = 0.089 + \frac{-\ 1.883\ \times 6.528}{-\ 67.115} =$$

$$0.089 + 0.186 = 0.275$$

8. Three doublets combined, see diagram.



Line of vertices

Line of lens nodals

Line of doublet nodals

Nodals of first combination

Nodals of second combination

First lens of 1*st doublet.* $r = \infty,\ s = 1,\ e = \frac{1}{2},\ \mu = \frac{8}{5}$

Whence $f = \infty,\ f' = -\frac{8}{3},$

$$A\,N = \frac{5}{8}\cdot\frac{\frac{1}{2}\cdot\infty}{\infty + \frac{-\ 8}{3} - \frac{1}{2}} = \frac{5}{8}\cdot\frac{\frac{1}{2}\cdot 1}{1 + 0 - 0} = \frac{5}{16}$$

$$A'\,N' = \frac{-\ \frac{1}{2}\cdot 5\ (-\ \frac{8}{3})}{8\ (\infty + f' - e)} = 0$$

$$F = \frac{5}{8}\cdot\frac{\infty\ (-\ \frac{8}{3})}{\infty - \frac{8}{3} - \frac{1}{2}} = \frac{5}{8}\cdot\frac{\frac{-\ 8}{3}}{1 - 0 - 0} = -\frac{5}{3}$$

Second lens of 1st doublet. $r = 1,\ s = -1,\ e = 1,\ \mu = \dfrac{3}{2}$

Whence $\qquad f = \dfrac{3}{2} \cdot 2 \cdot 1 = 3$

$$f' = \frac{-3}{2} \cdot (-1) \frac{2}{1} = 3$$

$$F = \frac{2 \cdot 3 \cdot 3}{3\,(3 + 3 - 1)} = \frac{6}{5}$$

$$A\,N = \frac{2 \cdot 1 \cdot 3}{3\,(3 + 3 - 1)} = \frac{2}{5}$$

$$A'\,N' = \frac{-1 \cdot 3 \cdot 2}{3 \cdot 5} = -\frac{2}{5}$$

First doublet. $f_1 = -\dfrac{5}{3},\ f_2 = \dfrac{6}{5},\ \epsilon = \dfrac{2}{5}$

$$F = \frac{-\frac{5}{3} \cdot \frac{6}{5}}{-\frac{5}{3} + \frac{6}{5} - \frac{2}{5}} = 2.3077$$

$$A\,N + \mathfrak{N} = \frac{5}{16} + \frac{\frac{2}{5}\left(-\frac{5}{3}\right)}{-\frac{5}{3} + \frac{6}{5} - \frac{2}{5}} = \frac{5}{16} + \frac{10}{3} = 1.08173$$

$$A'\,N' + \mathfrak{N}' = -\frac{2}{5} + \frac{-\frac{2}{5} \cdot \frac{6}{5} \cdot 15}{-13} = -\frac{2}{5} + \frac{36}{65} = 0.15385$$

First lens of 2d doublet. $r = \infty,\ s = 4,\ e = \dfrac{2}{3},\ \mu = \dfrac{8}{5}$

Whence $\qquad f = \dfrac{8}{5} \cdot \infty = \infty$

$$f' = -\frac{8}{5} \cdot \frac{4}{3} \cdot 5 = -\frac{32}{3}$$

$$F = \frac{5 \cdot \infty\left(-\frac{32}{3}\right)}{8\left(\infty - \frac{32}{3} - \frac{2}{3}\right)} = \frac{-5 \cdot 32}{3 \cdot 8} = \frac{-20}{3}$$

$$A\,N = \frac{2}{5} \cdot \frac{5}{8} \cdot \frac{\infty}{\infty - \frac{32}{3} - \frac{2}{3}} = \frac{2}{3} \cdot \frac{5}{8} = \frac{5}{12}$$

$$A'N' = \frac{-2}{3} \cdot \frac{5}{8} \cdot \frac{-32}{\infty - \ldots} = 0$$

Second lens of 2d doublet. $r = 4,\ s = -4,\ e = \frac{4}{3},\ \mu = \frac{3}{2}$

Whence $$f = \frac{3}{2} \cdot \frac{4}{1} \cdot 2 = 12$$

$$f' = \frac{-3}{2} \cdot \frac{-4}{1} \cdot 2 = 12$$

$$F = \frac{2}{3} \cdot \frac{12 \cdot 12}{12 + 12 - \frac{4}{3}} = \frac{72}{17}$$

$$A\,N = \frac{4}{3} \cdot \frac{2}{3} \cdot \frac{12 \cdot 3}{68} = \frac{8}{17}$$

$$A'N' = \frac{-4}{3} \cdot \frac{2}{3} \cdot 12 \cdot \frac{3}{68} = -\frac{8}{17}$$

Second doublet. $f_1 = -\frac{20}{3},\ f_2 = \frac{72}{17},\ \epsilon = \frac{8}{17}$

Whence $$F = \frac{-20}{3} \cdot \frac{72}{17} \cdot \frac{1}{\frac{-20}{3} + \frac{72}{17} - \frac{8}{17}} = 9.73$$

$$A\,N + \mathfrak{N} = \frac{5}{12} + \frac{\frac{8}{17} \cdot \frac{-20}{3}}{\frac{-20}{3} + \frac{72}{17} - \frac{8}{17}} = \frac{5}{12} + \frac{40}{37} = 1.498$$

$$A'N' + \mathfrak{N}' = \frac{-8}{17} + \frac{6 \cdot 72}{17 \cdot 37} = -.4705 + .6868 = 0.2163$$

First lens of 3d doublet. $r = \infty,\ s = 10,\ e = \frac{3}{4},\ \mu = \frac{8}{5}$

Whence $$f = \infty,\ f' = \frac{-8}{5} \cdot \frac{10}{3} \cdot 5 = \frac{-80}{3}$$

$$F = \frac{5}{8} \cdot \infty \cdot \frac{-80}{3} \cdot \frac{1}{\infty - \frac{80}{3} - \frac{3}{4}} = \frac{-5 \cdot 10}{3} = -\frac{50}{3}$$

$$A\,N = \frac{3}{4} \cdot \frac{5}{8} \cdot \frac{\infty}{\infty - \ldots} = \frac{15}{32}$$

$$A'\,N' = \frac{-3}{4} \cdot \frac{5}{8} \cdot \frac{-80}{3} \cdot \frac{1}{\infty - \ldots} = 0$$

Second lens of 3d doublet. $r = 10,\ s = -10,\ e = \frac{3}{2},\ \mu = \frac{3}{2}$

Whence $$f = \frac{3}{2} \cdot \frac{10}{1} \cdot 2 = 30$$

$$f' = \frac{-3}{2} \cdot \frac{-10}{1} \cdot 2 = 30$$

$$F = \frac{2}{3} \cdot \frac{30 \cdot 30}{30 + 30 - \frac{3}{2}} = \frac{400}{39}$$

$$A\,N = \frac{3}{2} \cdot \frac{2}{3} \cdot \frac{30}{30 + 30 - \frac{3}{2}} = \frac{20}{39}$$

$$A'\,N' = \frac{-3}{2} \cdot \frac{2}{3} \cdot \frac{30 \cdot 2}{117} = \frac{-20}{39}$$

Third doublet. $f_1 = \frac{-50}{3},\ f_2 = \frac{400}{39},\ \epsilon = \frac{20}{39}$

Whence $$F = \frac{-50}{3} \cdot \frac{400}{39} \cdot \frac{1}{-\frac{50}{3} + \frac{400}{39} - \frac{20}{39}} = \frac{5.400}{81} = 24.6923$$

$$A\,N + \mathfrak{N} = \frac{15}{32} + \frac{\frac{-20}{39} \cdot \frac{50}{3}}{-\frac{50}{3} + \frac{400}{39} - \frac{20}{39}} = 0.46875 + 1.23456 = 1.70331$$

$$A'N' + \mathfrak{N}' = \frac{-30}{39} + 0.75975 = -0.51282 + 0.75975 = 0.24693$$

Combination of second and third doublet, so that the posterior nodal point of the second doublet coincides with the anterior face of the third doublet.

This makes $\epsilon = 1.70331$, $f_1 = 9.73$, $f_2 = 24.6923$

Whence $$F = \frac{9.73 \times 24.6923}{9.73 + 24.6923 - 1.70331} = \frac{240.253}{32.7190} = 7.343$$

$$AN + \mathfrak{N} = 1.498 + \frac{1.70331 \times 9.73}{32.7190} = 1.498 + 1.277 = 2.775$$

$$A'N' + \mathfrak{N}' = 0.24693 + \frac{-1.70331 \times 24.6923}{32.7190} = 0.247 - 1.285 = -1.038$$

Combination of first doublet with preceding combination, so taken that the posterior nodal plane of the doublet coincides with the anterior face of the second doublet.

This makes $\epsilon = 2.775$, $f_1 = 2.3077$, $f_2 = 7.343$

Whence $$F = \frac{2.3077 \times 7.343}{2.3077 + 7.343 - 2.775} = \frac{16.948}{6.976} = 2.439$$

$$AN + \mathfrak{N} = 1.08173 + \frac{2.775 \times 2.3077}{2.3077 +} = 1.082 + \frac{6.405}{6.976} = 1.082 + .918 = 2.000$$

$$A'N' + \mathfrak{N}' = -1.038 + \frac{-2.775 \times 7.343}{2.3077 +} = -1.038 - \frac{20.377}{6.976} = -1.038 - 2.921 = -3.959$$

9. Camera lens, composed of three lenses, light from left.

First lens. $\mu = 1.6103$, $r = 1.264$, $s = 1.48$, $e = 0.105$, air space $= 0.232$.

Second lens. $\mu = 1.6103$, $r = -2.09$, $s = -0.553$, $e = 0.358$, air space $= 0.0053$.

Third lens. $\mu = 1.524$, $r = -0.5325$, $s = -2.8$, $e = 0.110$.

First lens. $f_1 = \dfrac{1.6103 \times 1.264}{0.6103} = 3.3351$

$$f_2 = \frac{-1.6103 \times 1.48}{0.6103} = -3.9050$$

$$F = \frac{-3.3351 \times 3.9050}{1.6103 \times (-0.6749)} = 11.9837$$

$$A\,N = \frac{3.3351 \times 0.105}{1.6103\,(-0.6749)} = -0.3222$$

$$A'\,N' = \frac{-(-3.9050) \times 1.05}{1.6103\,(-0.6749)} = -0.3773$$

Second lens.

$$f_1 = \frac{1.6103 \times (-2.09)}{0.6103} = -5.5145$$

$$f_2 = \frac{-1.6103\,(-0.553)}{0.6103} = +1.4591$$

$$F = \frac{-5.5145 \times 1.4591}{1.6103\,(-4.4134)} = 1.1322$$

$$A\,N = \frac{-5.5145 \times 0.358}{1.6103\,(-4.4134)} = 0.2778$$

$$A'\,N' = \frac{-1.4591 \times 0.358}{1.6103\,(-4.4134)} = 0.0735$$

Third lens.

$$f_1 = \frac{1.524\,(-0.5325)}{0.524} = -1.5487$$

$$f_2 = \frac{-1.524\,(-2.8)}{0.524} = 8.1435$$

$$F = \frac{-1.5487 \times 8.1435}{1.524 \times 6.4848} = -1.2762$$

$$A\,N = \frac{-1.5487 \times 0.110}{1.524 \times 6.4848} = -0.0173$$

$$A'\,N' = \frac{-8.1435 \times 0.110}{1.524 \times 6.4848} = -0.09064$$

First combination, of 2d and 3d lens.

$$f_1 = 1.1322,\ f_2 = -1.2762,\ \epsilon = -0.0854$$

$$A\,N = 0.2778,\ A'\,N' = -0.0906$$

Therefore $A\,N + \mathfrak{N} = 0.2778 + \dfrac{1.1322\,(0.0854)}{-0.0586} =$

$$0.2778 + 1.6499 = 1.9277$$

$$A'\,N' + \mathfrak{N}' = -0.0906 + \frac{-(-1.2762)\,(-0.0854)}{-0.0586} =$$

$$-0.0906 + 1.8598 = 1.7692$$

$$F = \frac{1.1322\,(-1.2762)}{-0.0586} = 24.656$$

Second combination, of 1st lens and 1st combination.

$$f_1 = 11.9837,\ f_2 = 24.6561,\ \epsilon = 2.5369$$

$$A\,N = -0.3222,\ A'\,N' = 1.7692$$

Therefore $A\,N + \mathfrak{N} = -0.3222 + \dfrac{11.9837 \times 2.5369}{34.1029} =$

$$-0.3222 + 0.8915 = 0.5693$$

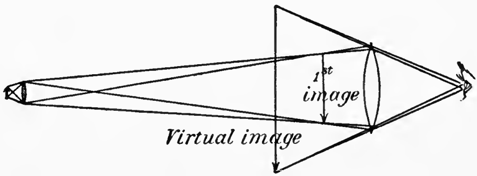
$$A'\,N' + \mathfrak{N}' = 1.7692 + \frac{-24.6561 \times 2.5369}{34.1029} =$$

$$1.7692 - 1.8345 = -0.0653$$

$$F = \frac{11.9837 \times 24.6561}{34.1029} = 8.664$$

Excellent for care in signs. Look out for negative ϵ, caused by the crossed nodals. To calculate ϵ, locate the nodals on a rough diagram.

108. Magnifying Power of a Microscope (Compound).



$$\text{Magnifying power} = \frac{i}{o} \cdot \frac{I}{i} = \text{1st magnif.} \times \text{2d magnif.}$$

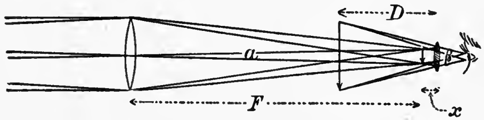
$$= \frac{v}{u}\left(1 + \frac{D}{f}\right) \qquad (\S 55)$$

f = focal length of the eyepiece
D = least distance of distinct vision
v = dist. from objective to image
u = dist. from objective to object
o = size of object
i = size of real image
I = size of virtual image

109. N. B.—In the microscope the magnifying power is the ratio between the virtual image and the object, because both are seen at the same distance, the distance of distinct vision. In the telescope, however (see next section), the virtual image and object are not seen at the same distance and the comparison must be made on a different basis; viz., comparison of the angles under which the virtual image and the object are seen. The distance at which the virtual image is to be considered depends upon the "set" of the eye of the observer. A person with a very flexible eye can vary the distance from far to near, which

produces a slight variation in the angle. A far-sighted or presbyopic eye has the eye "set" for the far distance, and therefore for the slightly smaller angle subtended by the virtual image, which, however, is practically the same angle. Ratio of angles could have been used in the microscope also.

110. Magnifying Power of a Telescope.



Angle under which object would be seen by the naked eye $= \alpha$, practically.

Angle under which object would be seen by the telescope $= \beta$.

Therefore, magnification $= \frac{\beta}{\alpha} = \frac{\mathrm{F}}{\mathrm{x}}$ approximately, since the angles are small.

But (§ 36) $\frac{1}{-\mathrm{D}} - \frac{1}{-\mathrm{x}} = \frac{1}{+\mathrm{f}}$ or $\frac{1}{\mathrm{x}} = \frac{\mathrm{D}+\mathrm{f}}{\mathrm{Df}}$

and $\frac{\mathrm{F}}{\mathrm{x}} = \frac{\mathrm{F}}{\mathrm{f}} \cdot \frac{\mathrm{D}+\mathrm{f}}{\mathrm{D}} = \text{mag.} = \frac{\mathrm{F}}{\mathrm{f}}$ nearly

For the eye looking at a landscape, $\frac{D+f}{D}$ is approximately 1, and the magnification $= \frac{F}{f}$

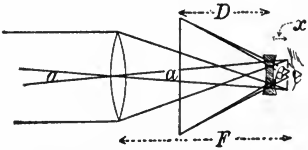
D = distance of distinct landscape vision
f = focal length of the eyepiece
F = focal length of the object of glass

Example. — Object glass of telescope is 20 ft. focal length. With a $\frac{1}{2}$ inch eyepiece, what is the magnification?

Ans. Mag. = 480.

111. Magnifying Power of Opera Glass (Galilean telescope).

α = angle of object at eye of observer, practically
β = angle of image
D = least distance of distinct vision
F = focal length of object glass
f = focal length of eyepiece



Magnification $= \dfrac{\beta}{\alpha}$ (practically exact for distant objects)

$$= \frac{F}{x} = \frac{F}{f} \cdot \frac{D - f}{D} = \frac{F}{f} \text{ nearly}$$

$$\left[\frac{1}{-D} - \frac{1}{x} = \frac{1}{-f} \quad \text{See § 37}\right]$$

Example. — If the object glass is 4 in. focus, and the eyepiece $1\frac{1}{2}$ in., what will be the magnifying power and the distance between the lenses?

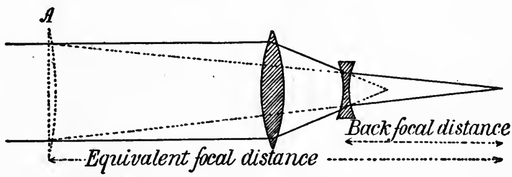
Ans. 8/3; 2.5.

112. Magnifying Power of Camera. — (See §§ 47, 53.) For telephoto camera, see § 120.

CHAPTER V

TELEPHOTO LENS

113. In § 29 we found that a negative lens interposed in the path of converging rays so that the aerial object fell within the focal distance of the negative lens gave a real image *beyond* the aerial object.



This is the principle of the telephoto lens, the aerial object being the real image made by the camera lens. As shown in the diagram, the result of interposing the negative lens is to give an image as if made by a long focus lens in the position A. But a long focus lens gives a large image, and usually requires a long camera box; i.e. the long back focal distance. By reference to the diagram it will be seen, however, that in the case of the telephoto lens, the back focal distance is very much less than the focal distance. Hence instead of a very long focus lens with its correspondingly long box, we have the combination of two lenses, a + and a −, and the same effect with a very much shorter box. See § 107, Ex. 2, § 93, Ex. 7.

114. Focal Length of a Telephoto Combination.

By §§ 94, 95, focal length $= F = \dfrac{-f_1 \cdot \mathrm{f}_2}{f_1 - \mathrm{f}_2 - \epsilon}$

$$\text{back focal distance} = \frac{-\mathrm{f}_2\,(f_1 - \epsilon)}{f_1 - \mathrm{f}_2 - \epsilon} = F_B$$

[f_1 = focal length of + lens
f_2 = of 2d lens
ϵ = distance between the nodes, emergence of 1st lens, incidence of 2d lens

115. Telephoto Magnification.

$\mathfrak{M}$ = increase of magnification due to combination as compared with the + lens alone

$$= \frac{\text{size of image made by combination}}{\text{size of image made by converging lens}}$$

$$= \frac{\text{focal length of combination}}{\text{focal length of converging lens}}$$

$$= \frac{F}{f_1} = \frac{f_1 f_2}{(f_1 + f_2 - \epsilon)\,f_1} = \frac{f_1 \mathrm{f}_2}{(f_1 - \mathrm{f}_2 - \epsilon)\,f_1} = \frac{-\mathrm{f}_2}{f - \mathrm{f}_2 - \epsilon}$$

$$= \frac{-\mathrm{f}_2\,(f_1 - \epsilon)}{\mathrm{f}_2\,(f_1 - \mathrm{f}_2 - \epsilon)} + 1 = \frac{+F_B}{\mathrm{f}_2} + 1$$

$$= \frac{\text{back focal distance}}{\text{num. val. of focal length of neg. lens}} + 1$$

Therefore F_B = back focal distance $= \mathrm{f}_2\,(\mathfrak{M} - 1)$

F = equivalent focus of combination

$$= \frac{F_B \cdot f_1}{\mathrm{f}_2} + f_1 = F_B \cdot m + f_1 \qquad \left[m = \frac{\mathrm{f}_1}{\mathrm{f}_2}\right.$$

Notice that F_B for a given $\mathfrak{M}$ is affected only by the negative lens used.

EXAMPLES

1. For $\mathfrak{M} = 3, f_2 = -3$, we get $F_B = 3\,(3 - 1) = 6$.
2. Rays forming a real image are intercepted by a con-

cave lens of 12 in. focal length at a distance 8 in. from the screen. How far must the screen be moved to be in focus again?

Ans. $\frac{1}{12} = \frac{1}{8} - \frac{1}{v}$ $\therefore v = 24, \therefore 24 - 8 = 16 =$ distance to be moved.

3. For $f_1 = 6, f_2 = -3$,

1. If $F_B = 12$ then $\mathfrak{M} = 5$ $F = 30$.
2. If $\mathfrak{M} = 3\frac{1}{2}$ then $F_B = 7\frac{1}{2}$ $F = 21$.
3. If $F_B = 7\frac{1}{2}$ then $F = 21$.

4. Dallmeyer's Telephotographic lens. $f_1 = 6, f_2 = -3$.

$$A\mathfrak{N} = \frac{6\epsilon}{3 - \epsilon}, \; A'\mathfrak{N}' = \frac{-3\epsilon}{3 - \epsilon}, \; F = \frac{-18}{3 - \epsilon}.$$

Since F must be $+$, $\epsilon > 3$, say $3\frac{3}{4}$.

Hence $A\mathfrak{N} = -30$, $A'\mathfrak{N}' = -15$, $F = 24$, $F_B = 9$.

5. In Ex. 5, suppose $F_B = 12$, what should ϵ be, and what will be the value of F?

Ans. $\epsilon = 3\frac{3}{5}$, $F = 30$.

6. $f_1 = 9\frac{1}{2}, f_2 = -13$.

Ans. $A\mathfrak{N} = -5\frac{1}{15}$, $A'\mathfrak{N}' = -6\frac{14}{15}$, $F = 16\frac{7}{15}$, $F_B = 9\frac{8}{15}$.

7. $f_1 = -13, f_2 = 9\frac{1}{2}$.

Ans. $A\mathfrak{N} = 6\frac{14}{15}$, $A'\mathfrak{N}' = \ldots$, $F = 16\frac{7}{15}$, $F_B = 21\frac{8}{15}$.

Notice that this is the lens of Ex. 6 turned around, and observe the large increase of F_B when the negative lens is in front.

116. Focal Distances.

$$\mathbf{F} = \frac{\mathbf{f}_1\mathbf{f}_2}{f_1 - f_2 - \epsilon} = \frac{\mathbf{f}_1\mathbf{f}_2}{d} = \frac{1}{m}\frac{f_1^2}{d} = m\frac{\mathbf{f}_2^2}{d} \qquad \left[m = \frac{f_1}{-f_2} = \frac{\mathbf{f}_1}{\mathbf{f}_2} \right.$$

$$\overline{Ff_1} = \overline{FN_1} - \overline{AN_1} - \overline{f_1A}$$

[$\overline{Ff_1}$ means the distance between F and f_1, etc.

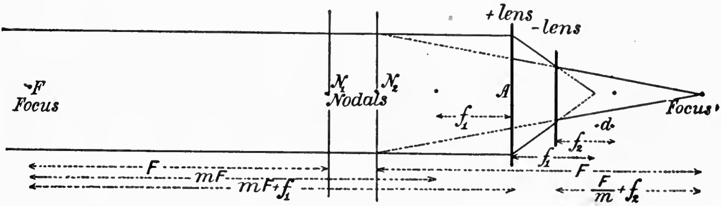
$$= \frac{f_1f_2}{d} - \frac{ef_1}{d} - \frac{f_1d}{d}$$

$$= \frac{f_1f_2 - ef_1 - f_1(f_1 + f_2 - \epsilon)}{d}$$

$$= \frac{-f_1^2}{d} = \frac{f_1^2}{\mathbf{d}}$$

$= mF =$ distance of front focal point of the combination from the front focal point of the positive lens

($-\overline{AN_1}$, because AN_1 is essentially negative, and FN_1 essentially positive, and the $-$ sign is needed in order to give them the same combinative sense)



Similarly $\frac{f_2^2}{\mathbf{d}} = \frac{F}{m} =$ distance of back focal point of the combination from the back focal point of the negative lens

Hence Distance between front focal point of the system and the front lens (nodal point) $= mF + f_1$

Distance between back focal point of the system and the negative lens $= \frac{F}{m} + f_2 = \frac{F}{m} - \mathbf{f}_2$

117. For a single lens, § 47

$$v = \frac{f}{N} + f = \text{dist. to image}$$

$$u = -(Nf + f) = \text{dist. to object}$$

$$\left[\begin{array}{l} \dfrac{1}{N} = \text{magnification} = \dfrac{v}{f} - 1 \\ N = \text{reduction factor} \end{array}\right.$$

Similarly for the telephoto combination

$$I = \text{dist. of image from neg. lens (nodal point)}$$

$$= \frac{F}{N} + \frac{F}{m} + f_2 = E$$

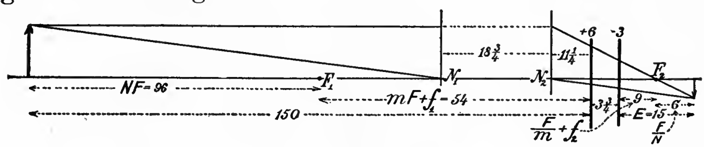
$$= \frac{F}{N} + \frac{F}{m} - \mathbf{f}_2 \qquad \text{§ 116}$$

$$O = \text{dist. of object from pos. lens (nodal point)}$$

$$= -(NF + mF + f_1) \qquad \text{§ 116}$$

$$\left[\begin{array}{l} N = \text{reduction factor for the system} \\ F = \text{focal length of the system} \\ f_1 = \text{focal length of + lens} \\ f_2 = \text{focal length of − lens} \\ m = \dfrac{\mathbf{f}_1}{\mathbf{f}_2} = \dfrac{f_1}{-f_2} \\ E = \text{bellows extension} \end{array}\right.$$

Example.—$f_1 = 6$, $f_2 = -3$, $\epsilon = 3\frac{3}{4}$. The results are given in the diagram.

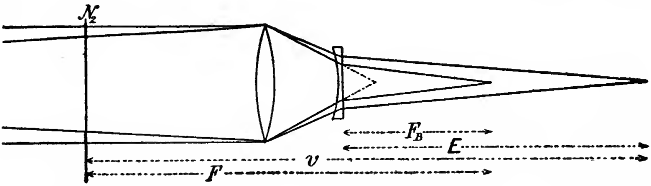


118. Focal Radius in Terms of the Magnification, $\frac{1}{N}$.

$$\frac{1}{N} = \frac{v}{F} - 1 \qquad \text{[§ 47}$$

Therefore $v = \left(\dfrac{1}{N} + 1\right) F$

But $\quad v - F = E - F_B$ (See diagram)



therefore $F_B = E - \dfrac{F}{N}$

But $\quad F = m F_B + f_1$ [§ 115

$$= m\left(E - \frac{F}{N}\right) + f_1$$

whence $\quad F = \dfrac{m\,E + f_1}{\frac{m}{N} + 1}$ $\quad\left[\begin{array}{l} m = \dfrac{\mathbf{f}_1}{\mathbf{f}_2} = -\dfrac{f_1}{f_2} \\ E = \text{bellows extension} \\ f_1 = \text{focal length of } + \text{ lens} \\ N = \text{reduction factor} \end{array}\right.$

If $N = \infty$ (i.e. object very distant), this becomes

$$F = mF_B + f_1, \text{ as before}$$

EXAMPLES

1. $f_1 = 10$, $f_2 = -4$, with the object 80 distant, to reduce to $\frac{1}{2}$ size.

Mag. produced by 1st lens $= \dfrac{10}{80 - 10} = \dfrac{1}{7}$ [See § 47

$\therefore \quad M =$ mag. of 1st image $= \frac{7}{2}$ $\quad [M \cdot \frac{1}{7} = \frac{1}{2}$

$\therefore \quad E = 4\left(\frac{7}{2} - 1\right) = 10$ [§ 47

$\therefore \quad F = \dfrac{\frac{10}{4} \cdot 10 + 10}{\frac{10}{4} \cdot \frac{1}{2} + 1} = 15\frac{5}{9}$

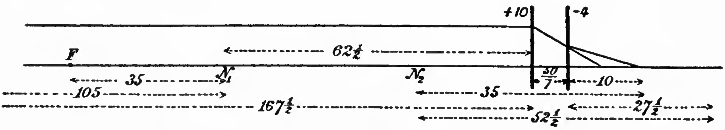
2. If $f_1 = 8$, $f_2 = -3$, and object is 60 in front, $N = 2$.

1st mag. $= \frac{2}{13}$, $M = \frac{13}{4}$, $F_B = \frac{27}{4}$, $F = 11\frac{1}{7}$

3. $f_1 = 10$, $f_2 = -4$, with the object $167\frac{1}{2}$ in front of the $+$ lens, to reduce to $\frac{1}{2}$ size.

$$M = \frac{63}{8} \qquad E = 27\frac{1}{2} \qquad F = 35$$

Other facts are shown in the diagram below.



$$A\mathfrak{N} = -62\frac{1}{2} \quad A'\mathfrak{N}' = -25 \quad F = 35 \quad F_B = 10$$

$$u = -105 \quad v = 52\frac{1}{2}$$

4. Positive lens of 8 in. focus and negative lens of 3 in. focus with the object 60 in. away, to reduce to $\frac{1}{2}$ size.

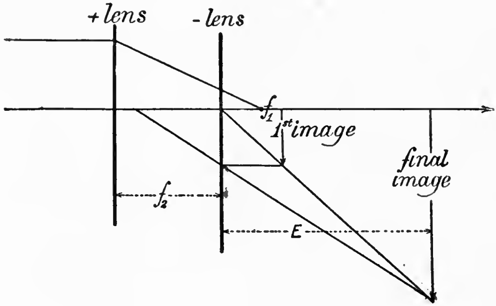
Mag. of $+$ lens $= \frac{2}{13}$, $M = \frac{13}{4}$, $E = \frac{27}{4}$, $F = \frac{78}{7}$

119. Distance to Object for a given Magnification, $\frac{1}{N}$.

M = magnif. of 1st image due to 2d lens

$$= \frac{\text{final image}}{\text{1st image}} = \frac{E + \mathrm{f}_2}{\mathrm{f}_2} \qquad \text{(Conf. § 47)}$$

$$E = \text{extension of bellows} = F_B + \frac{F}{N} \qquad \text{[§ 118}$$



$$F_B = \frac{F - f_1}{m} \qquad [\S\ 115$$

$$\frac{1}{n} \cdot M = \frac{1}{N} = \text{1st reduction} \times \text{mag. due to 2d lens} = \text{final reduction}$$

$$u = \text{dist. of object} = f_1\,(n + 1) \qquad \left[\frac{1}{n} = \frac{f_1}{u - f_1},\ \S\ 47\right.$$

Find F_B, then E, then M, then n, then u.

Example.—$F = 24, f_1 = 6, f_2 = -3, N = 5$

$\therefore \quad F_B = 9,\ E = 13\frac{4}{5},\ M = 5\frac{3}{5},\ n = 28$

$\therefore \quad u = 6\,(28 + 1) = 174$ in.

120. Reduction Factor.

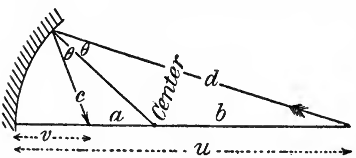
From § 118 $N = \dfrac{mF}{mE + f_1 - F}$. For N, see §§ 119, 117.

CHAPTER VI

REFLECTION AT SPHERICAL SURFACES

NOTE. — This chapter is introduced on account of some experimental observations.

121. The angles of incidence and reflection are equal. (See any text-book on physics.)



By geometry, a line bisecting the angle of a triangle divides the opposite side into segments proportional to the adjacent sides; hence, since the angle of reflection is equal to the angle of incidence

$$\frac{a}{b} = \frac{c}{d}$$

But for points near the axis, $d = u$, $c = v$, whence

$$\frac{a}{b} = \frac{v}{u}, \text{ or } \frac{r - v}{u - r} = \frac{v}{u}$$

or

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$$

For $u = \infty$, i.e. parallel rays from a distant object, $v_f = \frac{r}{2} = f$, called the focal distance, whence

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r} = \frac{1}{f}$$

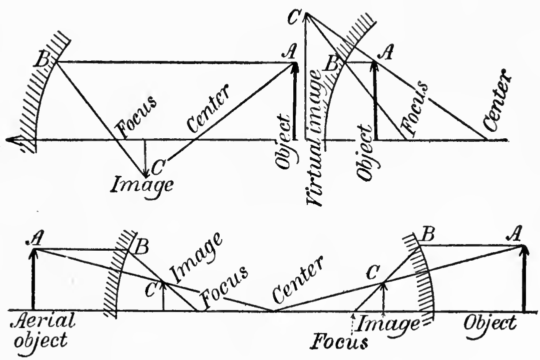
This is a general equation, applicable to convex or concave surfaces, attention being paid to the values of the letters, measurements to the right being positive.

122. In reflection, it simplifies the questions of signs if we suppose the light to come from the right, thus making u positive for real objects and negative for aerial objects, with the following tabulation of results.

			u	r	v
Divergent pencil Real object	concave mirror	$u > \frac{r}{2}$	+	+	+
		$u < \frac{r}{2}$	+	+	−
	convex mirror		+	−	−
Convergent pencil Aerial object	concave mirror		−	+	+
	convex mirror	$u < \frac{r}{2}$	−	−	+
		$u > \frac{r}{2}$	−	−	−

Negative v means the image is virtual.

123. Graphic Construction for the object in various positions; determined by known lines, focal and central.



The order of the letters denotes the order of construction.

Caution. — This is accurate only near the vertex, and is introduced here in order to illustrate the method. In actual construction, symbolize the surface by a straight line perpendicular to the axis, as was done in the case of

a thin lens. For tracing any ray, we have similarly to § 32, $rs \rightarrow \phi \parallel$ to c, where s = surface, c = line through center of curvature.

124. Magnification.

$$\text{Magnification} = \frac{\text{Image}}{\text{Object}} = \frac{v - r}{r - u} = \frac{v}{u}$$

EXAMPLES

1. A candle flame 1 cm. long, 36 cm. in front of a concave mirror, whose focal length is 30 cm., gives what image and where?

Ans. $\frac{1}{v} + \frac{1}{36} = \frac{1}{30}$ $\therefore v = 180.$ Mag. $= \frac{180}{36} = 5$

2. A flame 2 in. in front of a positive lens of 1 in. focus, and plane mirror $\frac{1}{2}$ in. behind the lens, reflects the rays back through the lens. Show that the real image will be $\frac{1}{2}$ in. from the lens. First image is 2 behind the lens or $1\frac{1}{2}$ behind the mirror. Second image (aerial) $1\frac{1}{2}$ before the mirror or 1 before the lens. Third image is $\frac{1}{2}$ before the lens.

3. How far from a concave mirror must an object be placed to be magnified n times?

Ans. $v = nu$ (real image); $v = -nu$ (virtual image). For real image, $\frac{1}{f} = \frac{1}{v} + \frac{1}{nu}$. $\therefore u = \frac{(n+1)f}{n}$. For virtual image, $\frac{1}{f} = \frac{1}{u} + \frac{1}{-nu}$ $\therefore v = \frac{(n-1)f}{n}$.

4. Gas flame 10 in. from wall. Required real image on the wall three times as large. What mirror and where?

Ans. $v = 3u$; u = dist. from mirror to object; $10 + u = 3u$. $\therefore u = 5$. $\frac{1}{f} = \frac{1}{5} + \frac{1}{15}$. $\therefore f = 3\frac{3}{4}$. Result. a concave mirror, $3\frac{3}{4}$ focal length, 5 in. from object.

5. $u = 10, f = -30.$

Ans. $v = -\frac{30}{4}$. Mag. $= -\frac{3}{4}$.

6. Object = 1 in., $u = 18, f = +15.$

Ans. $v = 90$. Mag. $= 5$.

7. $u = \frac{1}{2}f.$

Ans. $v = -f$. Image virtual = twice object

8. Mag. = 12. Object 11 from screen.

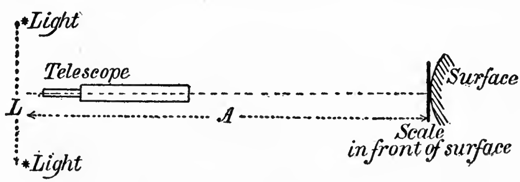
Ans. $12u - u = 11$. $\therefore u = 1$, $\therefore f = \frac{12}{13}$.

CHAPTER VII

EXPERIMENTAL OBSERVATIONS

To get thoroughly satisfactory results requires care, experience, and a trained eye. The average untrained eye cannot see things as they actually exist. Make several observations and take the mean of them.

125. To Find Radius of Curvature of a Surface.



The lights (or other suitable objects) produce two images, the distance apart of which is observed through the telescope on the scale.

$$r = \text{radius of curv.} = \frac{2\,Al}{L - 2\,l} \text{ (for convex surface)}$$

$$= \frac{2\,Al}{L + 2\,l} \text{ (for concave surface)}$$

l = distance of the images apart
L = actual dist. of the lights
A = dist. from surface to line of the lights

The less the curvature the greater A must be to get accurate results. In the case of biconcave or biconvex lenses it is easily seen which images are from the front or back surface, by their inverted or erect position.

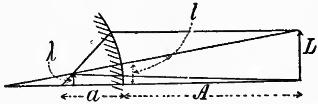
Proof. — The distance a, at which the virtual image of the line L is formed behind the surface, is given by (see § 121).

$$\frac{1}{a} = \frac{1}{A} + \frac{2}{r}, \quad \text{whence } a = \frac{Ar}{2A + r}$$

$$\frac{\lambda}{L} = \frac{r - a}{A + r}, \quad \frac{1}{\lambda} = \frac{A}{A + a} \qquad [\lambda = \text{length of image}$$

$$\text{whence } \frac{1}{L} = \frac{r}{2(A + r)}$$

$$\text{whence } r = \frac{2Al}{L - 2l}$$



Similarly for a concave surface.

(*Second method.*) Make object (small illuminated disc in screen) and image coincide by reflection from concave mirror, or by interposing a positive lens in front of the convex mirror.

For concave mirror, $r = 2f$ equals the distance of the mirror surface from the image (object).

For convex mirror, find image of disc when mirror is removed. The distance of this point from the front of the mirror before removal is r.

If object and image do not coincide, f can be calculated from the u and v distances. Check graphically by diagram 1 of § 40.

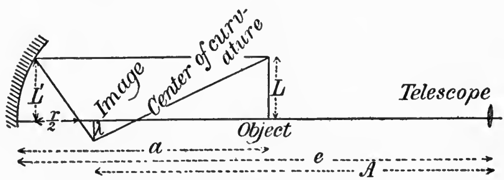
Or for a concave mirror f can be measured directly by first rendering the rays parallel by a positive lens; the distance of the image from the mirror is f.

126. By Matching with Surface of Known Curvature, using the scale on both as in § 125.

This matching can be roughly done by holding the surfaces in the hand, and observing the image of some bright object, window, lamp globe, etc.

127. Radius of Curvature of Surface of Small Curvature. Focus a telescope on a scale at a distance A from the object glass. With the telescope thus focussed, let the image of an object reflected from the surface to be tested be clearly seen when the distance between the object and the surface is a, and that between the surface and the object glass of the telescope is e. Then

$$\text{rad. of curv.} = r = 2\,a\,\frac{A - e}{A - e - a}$$



Proof. $\dfrac{\lambda}{L} = \dfrac{r - e + A}{a - r} = \dfrac{e - A - \frac{r}{2}}{\frac{r}{2}}$, L' being practically at the surface [Sim. triangles

Whence $$r = 2\,a\,\frac{A - e}{A - e - a}$$

The best value for e is about $\dfrac{A}{2}$.

Positive r denotes a concave surface; negative r a convex surface.

The absence of parallax between the cross wires of the telescope and the image is the test of distinct vision.

FOCAL LENGTH OF A THIN POSITIVE LENS

128. With the Sun. — Distance of the image of the sun from the lens $= f$.

129. Lens Distances.—$f = \frac{uv}{u - v}\cdot$ (See § 16.) Distances to the left are negative. u = distance from lens to object, v = distance to image. For accurate focussing, see § 156. Solve graphically by diagram 1 of § 40, for check.

130. With a Telescope. — Focus the telescope on some distant object. Place the lens in front of the object glass. Look through the telescope, without altering its length, at some plane object (a newspaper), adjusting the distance for distinct vision. The distance of the object from the lens = f, because the lens sends parallel rays into the telescope already set for parallel rays.

131. By Different Positions of the Lens.

$f = \frac{1}{4}\left(l - \frac{a^2}{l}\right)$. l = distance between image and object, a = distance between the two positions of the lens when giving a distinct image, the object and screen remaining fixed.

Proof. — The distances of object and image from the lens are $\frac{1}{2}(l + a)$ and $\frac{1}{2}(l - a)$, whence (§ 17),

$$\frac{1}{f} = \frac{2}{\mathrm{l} - \mathrm{a}} + \frac{2}{\mathrm{l} + \mathrm{a}} = \frac{4\,\mathrm{l}}{\mathrm{l}^2 - \mathrm{a}^2}$$

132. From Equality of Object and Image. — Distance between object and image = $4f$. (See § 37, Ex. 2.)

133. Comparison of Images. — A candle (or illuminated aperture) is placed a distance a from a screen and the image focussed on the screen. On moving the lens towards the candle another image is formed which is m times as large as the former.

$$\text{The focal length} = \frac{a\sqrt{m}}{(1 + \sqrt{m})^2}$$

Proof: By § 131, $f = \dfrac{a^2 - b^2}{4a}$

$\left[\begin{array}{l} b = \text{distance between} \\ 2 \text{ positions of the lens} \end{array}\right.$

By § 47 $\quad m = \dfrac{\dfrac{a - \dfrac{a-b}{2}}{f} - 1}{\dfrac{a-b}{2f} - 1} = \dfrac{a + b - 2f}{a - b - 2f}$

From the 1st eq. $\quad b^2 = a^2 - 4\,af$

From the 2d eq. $\quad b = \dfrac{(a - 2f)\,(m-1)}{m+1}$

$$\therefore\ b^2 = \frac{(a^2 - 4\,af + 4\,f^2)\,(m-1)^2}{(m+1)^2}$$

$$\therefore\ a^2 - 4\,af = \frac{(a^2 - 4\,af + 4\,f^2)\,(m-1)^2}{(m+1)^2}$$

$$\therefore\ f^2 + \frac{4\,amf}{(m-1)^2} = \frac{ma^2}{(m-1)^2}$$

$$f = \frac{-2\,am}{(m-1)^2} + \sqrt{\frac{ma^2}{(m-1)^2} + \frac{4\,a^2m^2}{(m-1)^4}}$$

$$= \frac{-2\,am}{(m-1)^2} + \frac{a}{m-1}\sqrt{m + \frac{4\,m^2}{(m-1)^2} = \frac{m\,(m^2 + 2m + 1)}{(m-1)^2}}$$

$$= \frac{-2\,am + a\,(m+1)\sqrt{m}}{(m-1)^2}$$

$$= \frac{a\sqrt{m}\,(m + 1 - 2\sqrt{m})}{(m-1)^2} = a\sqrt{m}\left(\frac{\sqrt{m} - 1}{m - 1}\right)^2$$

$$= a\sqrt{m}\left(\frac{\sqrt{m} - 1}{(\sqrt{m} - 1)\,(\sqrt{m} + 1)}\right)^2 = \frac{a\sqrt{m}}{(\sqrt{m} + 1)^2}$$

Q.E.D.

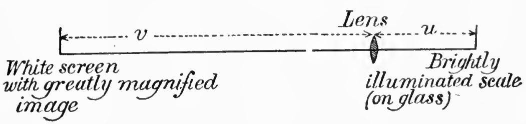
FOCAL LENGTH OF THICK POSITIVE LENS

134. From Highly Magnified Image.

$$f = v \frac{l}{L + l}$$

l = length of a division of the scale,
L = length of the image.

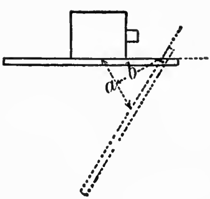
Proof: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$. (See § 16.) $\frac{L}{l} = \frac{v}{u}$. Whence $f = v \frac{l}{L + l}$. Since v is very large, small errors in its measurement or mistakes in locating the nodal point (to which it should be measured) do not materially affect the result.



Screen and scale may be interchanged, with diminished image, using a lens to read the image, in which case

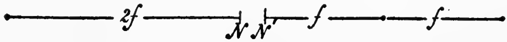
$$f = u \frac{L}{L + l}$$

135. Swing of Camera or Lens Carrier. — Swing the camera or lens carrier horizontally on a table (guide by a flat stick with a small nail through one end) until a distant vertical object is focussed on two vertical lines (short) on the extreme edges of the ground glass screen. Mark the angle of swing on the table. In the angle plot the distance a, between the lines on the screen, perpendicular to the bisector of the angle. The bisector, b, will equal the focal length.



a = distance between lines on the screen. b = focal distance

136. Movement of Screen. — Focus on very distant object. Then focus on a near object, making the image



and object equal (same size discs with parallel lines, one pattern covering the other). f = the distance the screen is moved. (Conf. § 37, Ex. 2; § 47, Ex. 15.)

137. Movement of Screen. — Focus for very distant object. Focus on a near object for the image $= \frac{s}{d}$ of object. (s = number of units on scale, d = number of units on screen covered by the s units of the scale.)

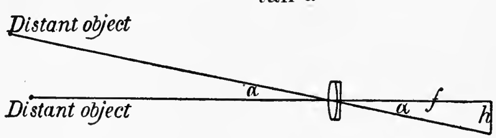
Then $\quad f = \dfrac{sa}{d}$ $\quad$ [a = distance moved by screen between the two focussings

Proof: $\quad f = \dfrac{v}{m+1}$ $\quad$ [v = dist. to image. See § 47
m = magnification

$$= \frac{f+a}{m+1}$$

Whence $f = \dfrac{a}{m} = \dfrac{sa}{d}$ $\quad$ [m = magnif. $= \dfrac{d}{s}$

138. Angle of Vision. $f = \dfrac{h}{\tan \alpha}$. (See § 2.) h can be



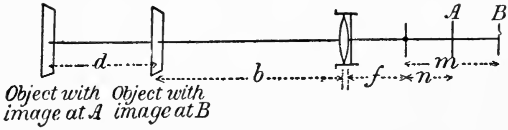
measured by a scale or a sliding lens. α must be measured by an instrument, or $\tan \alpha$ can be found by § 157.

139. Unit Screen Movement (Lionel Laurence)

$$f = \sqrt{\frac{dmn}{m-n}} = \sqrt{2d} \text{ if } m = 2n,\ n = 1$$

Proof: $\dfrac{1}{f+n} + \dfrac{1}{b+d} = \dfrac{1}{f}$, whence $b = \dfrac{f(f+n)}{n} - d$

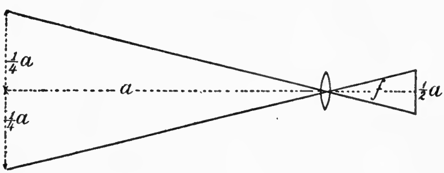
[§ 74



For B $\dfrac{1}{f+m} + \dfrac{1}{b} = \dfrac{1}{f}$, whence $b = \dfrac{f(f+m)}{m}$

whence $$f = \sqrt{\frac{dmn}{m-n}}$$

140. Measurement of Image. — At a distance a = at least $100f$, so as to rank as a distant object, set off at right angles two marks $\frac{1}{4}a$ distant from the center line.



The distance between the two images on the screen will be $\frac{1}{2}f$. This distance is most accurately measured by a sliding lens (microscope) focussed on the aerial image.

Any other submultiple of a can be similarly used.

If a is not large enough to be ranked as distant, then the distance apart of the images is $\frac{v}{2}$ instead of $\frac{f}{2}$. From this we can find f as follows:

$$f = \frac{av}{a+v} = v - \frac{v^2}{a-v} \qquad \left[\frac{1}{v} + \frac{1}{a} = \frac{1}{f}\right.$$

This result is accurate to within about 5 per cent.

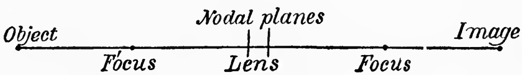
141. Comparison with Standard Lens, with distant object.

$$f = \frac{\text{focal length of standard} \times \text{length of image by lens}}{\text{length of image by standard lens}}$$

(See Ex. 16, § 47.)

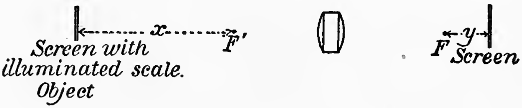
142. Double Focus, with Equal Distances to Object and Image.

Find F and F' for parallel rays. Place object and screen at distances somewhat less than $\frac{1}{2}$ the distance between F and F', and move by small equal increments until the object and image are equally distant from F and F' and



the image *distinct*. These distances will be the focal length required. (See § 47, Ex. 15, in connection with § 74.)

143. Double Focus, with Unequal Distances to Object and Image.



Determine F, F' for parallel rays, and then determine x and y for distinct image. Then

$$f = \text{focal length} = \sqrt{xy} \qquad \text{[See § 75}$$

If the principal focus is within the combination, interpose between object and combination any good short focus positive lens and adjust till the image is reflected back by a mirror to coincidence with the object (or slightly to one side by tilting the mirror). Then the image will be at the principal focus of the combination, since the emerging rays are parallel and reflected back parallel by the mirror. Remove the combination and find the image made by the positive lens. Determine this point (on the mounting) relative to

the combination before it was removed: it will be the principal focus (F) of the combination. Find F' in the same way. Then find x, y, as before, and $f=\sqrt{x\,y}$. (Microscope ocular.)

144. By Movement of Lens. — Same method as in § 131.

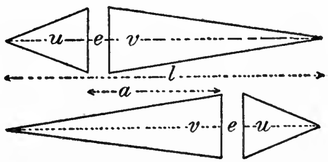


Diagram showing the extreme rays from a point; in the two positions. e = distance between nodals; a = distance between the two positions of the lens; l = distance between the screen and object

$$a = v - u$$

$$u + v + e = l$$

$$\because\ u = \frac{l - a - e}{2}, \quad v = \frac{l + a - e}{2}$$

$$\because\ \frac{1}{f} = \frac{2}{l + a - e} + \frac{2}{l - a - e} \qquad [\S\ 74$$

$$\therefore\ f = \frac{l^2 - a^2}{4\,l} - e\,\frac{l^2 + a^2}{4\,l^2}, \text{ very small terms being omitted}$$

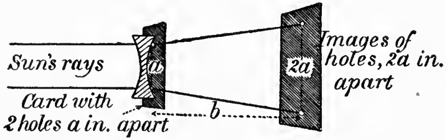
$$= \frac{l^2 - a^2}{4\,l} - \tfrac{1}{3}\,\frac{l^2 + a^2}{4\,l^2} \qquad [\S\ 67$$

Notice the correction induced by the thickness of the lens. (Conf. § 131.)

FOCAL RADIUS OF A NEGATIVE LENS

145. With Sun. — If lens is deep and not too small, focal length = b. (Conf. § 47, Ex. 14.)

This is an uncertain method, on account of the indistinctness of the bright patches.



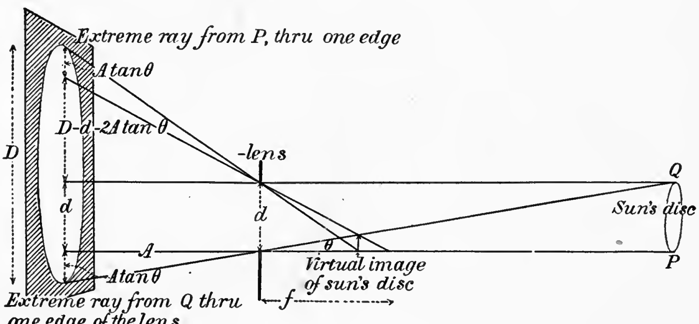
146. With Sun. $f = \dfrac{Ad}{D - d - 0.0094\,A}$, d = diameter of the lens aperture, D = diameter of the circle of light cast by the lens when in the path of the sun's rays, A = distance of the screen from the lens, $0.0094 = 2$ tan of the apparent diameter of the sun.

If the lens is deep and not too small, we can write, neglecting the $0.0094\,A$,

$$f = \frac{Ad}{D - d}$$

which becomes $f = A$, if $D = 2\,d$, as in § 145.

Proof.



By similar triangles $\dfrac{f}{d} = \dfrac{A}{D - d - 2\,A\tan\theta}$

147. With Stronger Positive Lens. — Combine the two lenses and find focus F. Find focal length of positive lens, F'. Then (§ 91)

$$f = \text{focus of negative lens} = \frac{F\,F'}{F' - F}$$

The positive lens should be so chosen as to make the difference $F' - F$ as large as possible.

148. With Positive Lens and Comparison of Images. — Focus with a positive lens and measured image. Interpose negative lens and measure new image. Call the magnification over the positive image, M.

Move the negative lens a small distance, D, nearer the screen, and measure the image, calling its magnification over the positive image, M'.

Then focal length of neg. lens $= f = \dfrac{D}{M' - M}$

Proof: $\quad \dfrac{d}{f} + 1 = M, \quad \dfrac{d'}{f} + 1 = M'$

Therefore, since identically $\quad \dfrac{d' - d = D}{\dfrac{d'}{f} + 1 - \left(\dfrac{d}{f} + 1\right)} = f$

then $$f = \frac{D}{M' - M}$$

d, d' = distances from negative lens to screen.

If we move the combination instead of the negative lens, and call M', M the actual magnifications of the images for two positions of the object, this method will apply to a microscope objective, or to the ocular (positive or negative) by inverting it.

149. With Positive Lens and Comparison of Images (Lindsay Johnson method).

Focus with positive lens and measure image.

Interpose negative lens and measure new image, adjusting

until the magnification over the positive image is 2; call the distance between screen and negative lens, a.

Move the negative lens until the magnification over the positive image is 3. Call the distance between the negative lens and the screen, b.

Then F = focal radius of the negative lens $= b - a$.

Proof: By preceding case

$$F = \frac{b - a}{M' - M} = \frac{b - a}{3 - 2} = b - a$$

TO LOCATE THE NODAL POINTS

150. (a) Determine the focal radius (§§ 134–144) and lay off this distance from the focal point, marking the result on the mounting. (Conf. § 57.) This is the node of emergence.

(b) Locate the point (by twisting the lens around a vertical axis: on the optical bench) around which the lens can be turned on a vertical axis without displacing the image. This is the node of emergence.

If the nodal of emergence is beyond the center of rotation, the image will move in a contrary direction to that of the back of the lens, and vice versa.

This point can also be determined by reflecting the image back through the lens to coincidence with the object by means of a mirror (or slightly to one side by tilting the mirror). When a slight movement of rotation produces no movement of the reflected image the axis of rotation is at the nodal, and moreover the focus of the lens is the distance of the axis of rotation from the object (image).

Reverse the lens and repeat the operation, to find the other nodal: both cases.

MAGNIFYING POWER: TELESCOPE

151. Visual Comparison of Images. — Distant object.

$$\text{Mag. power} = \frac{N}{n}$$

[N = number of clapboards (divisions on a scale) seen with one eye (naked) which are covered by n clapboards seen with the other eye through the telescope

152. Visual Comparison of Images. — Near object.

Focus the telescope on a very distant object, and then fix in front of the object glass a thin convex lens of low power (a spectacle glass of about 2 m. focal length).

The telescope is then pointed to a scale at such a distance that the divisions appear well defined, focussing by moving the scale, not the telescope tube length.

$$\text{Mag. power} = \frac{N}{n} \cdot \frac{a}{b}$$

[N, n = same as in § 151
a = dist. from scale to object glass of telescope
b = dist. from scale to eye of observer

Proof. — With the lens in front of the telescope, we have practically a large microscope, in which the magnification is

$$\frac{b+f}{f} \cdot \frac{F}{a} \quad (\S\ 108)$$

[b = dist. at which the scale is seen
f = focal length of eyepiece
F = focal length of object glass
a = focal length of intruded lens

But the magnification of the telescope is (§ 110)

$$\frac{b+f}{f} \cdot \frac{F}{b} \qquad \left[b = \text{distance of distinct landscape vision} \right.$$

To convert the first into the second we must multiply by $\frac{a}{b}\cdot$

Q.E.D.

MAGNIFYING POWER OF A MICROSCOPE

153. Visual Comparison of Images. — With one eye (naked) count the divisions on a scale 10 in. (25 cm.) from the eye which are covered by one or more divisions on a scale seen through the microscope.

$$\text{Mag. power} = \frac{N}{n} \qquad \left[\begin{array}{l} N = \text{number of divisions seen by the naked eye at 10 in.} \\ n = \text{number of divisions seen in the same space through the microscope} \end{array} \right.$$

A convenient way for observing N is to use a camera lucida on the eyepiece, with the naked-eye scale 10 in. from the eye, through the camera lucida.

154. To find the Work done by the Ocular. — Focus on the object with the ocular in place. Remove the ocular and with the aid of a small lens and piece of ground glass (see § 156) find the position of the real image made by the objective. Measure its distance from the top of the tube, and note the corresponding place on the ocular. Having previously determined the focal length of the eye and field lenses of the ocular, calculate (graphically or algebraically) the reduction caused by the field lens. (The position of this reduced image should be just inside the focus of the eye lens. See diagram of § 37.) Multiply this by the magnification caused by the eye lens. This product is the final action of the ocular.

If the first focussing is done on a scale, on or in the position of the ground glass, it will give the magnification caused by the objective.

The product of the two magnifications should equal that found by § 153.

(*Second method.*) Place a small rectangular opening of known width at one end of a tube 10 in. long, in the other end of which is placed the ocular to be measured.

At the *image* of the rectangular opening (slightly above the ocular) place a scale and with a lens read the width of the image on the scale. The reduction from rectangular opening to image, inverted, will be the magnification of the ocular.

$$\text{Mag. power} = \frac{\text{width of rectangular opening}}{\text{width of image on the scale}}$$

Proof.—The details are left to the reader, with the following guide. Make a diagram (skeleton, §§ 25, 77) of the ocular and by § 61, illustrated by § 95, construct the nodal lines. Starting with the virtual image (call it A) 10 inches to the left of the right-hand lens of the ocular, derive from it the aerial object (call it B), just outside the lens, which produced the virtual image. The ratio of magnification between these will be the magnifying (reduction by first lens, and final magnification by second lens) power of the ocular. For, a real object, the rectangle, at A, since these are conjugate points, will give a real image at B, which can be measured by a scale, and the ratio between this A and B will be the same as before.

Notice, due to the crossing of the nodals, the aerial object B gives a *virtual* image at A, since B is to the left of the H nodal line and within the F focus, though outside the lens. (See § 95, Ex. 4.)

When the objective is in operation the aerial object is generally within the ocular combination, instead of outside, and the virtual image at infinity. But this does not materially alter the angle under which the virtual image is seen, and therefore not the magnifying power.

Check this by locating A and B through the *lenses*, instead of through the nodal planes. Pass A toward the left to infinity, and note how B passes through the lens to a point between the lenses; the point to which the image made by the objective is deflected by the first lens of the ocular. The aerial image at this point gives the virtual image at infinity seen by the eye of the observer.

See also method of § 143.

155. To Find the Index of Refraction. — Sight the objective of a microscope on a well-marked, hard (celluloid) white surface, or piece of scratched glass, and read the scale on the limb of the microscope (vernier attachment necessary), the aperture of the objective having been diminished by slipping over the objective a cap (paper) with a small central opening (1 mm. = $\frac{1}{25}$ in. diameter).

Having marked the center of the lens whose index of refraction is desired, by a small circular ink spot with a clear center on the upper surface (in order to locate the center through the microscope), slip the lens under the objective and focus on the center of the upper surface. The difference of the readings is the thickness (t) of the lens.

Then carefully focus, through the lens, on the marked white surface on which the lens rests. The difference of the last two readings will be a, $t - a$ being the amount the objective must be raised from focussing on the white surface, due to the interposition of the lens. Then

$$\mu = \frac{t}{a} \cdot \frac{r - a}{r - t}$$

or if the lens is merely plane glass

$$\mu = \frac{t}{a}$$

μ = index of refraction
t = thickness of the lens
a = amount the objective must be lowered from focussing on the top of the lens, to bring the mark on the white surface into focus
r = radius of curvature of the upper surface of the lens

N.B. — To get satisfactory results, very great care and a fine vernier are necessary.

Proof.

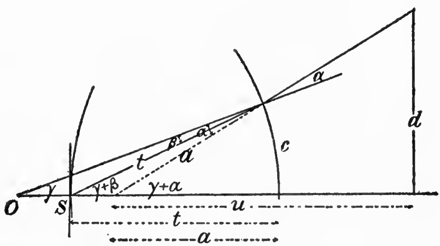


Diagram showing the lens in place under the objective. O = center of curvature of the upper surface. S = marked spot on the white surface on which the lens rests. a = distance the stage must be raised or the objective lowered in focussing from the top of the lens to the marked spot S (through the lens). α = angle of incidence, β, of refraction. d = semi-opening of lens. c = semi-diameter of visible part of lens.

Note. — α, β, and γ are supposed to be so small (due to the narrowed opening of the objective) that their cosines = 1 and their sines = α, β, γ, respectively, as also their tangents = α, β, γ. (See any trigonometry, Functions of Small Angles.)

$$\frac{\sin(\gamma + \alpha)}{\sin(\gamma + \beta)} = \frac{t}{a}$$

$$\therefore \quad \frac{\sin\gamma\cos\alpha + \cos\gamma\sin\alpha}{\sin\gamma\cos\beta + \cos\gamma\sin\beta} = \frac{t}{a} = \frac{\frac{c}{r}\cos\alpha + \sin\alpha}{\frac{c}{r}\cos\beta + \sin\beta}$$

$$\left[\sin\gamma = \frac{c}{r}\right.$$

$$= \frac{\frac{c}{r} + \sin\alpha}{\frac{c}{r} + \sin\beta} \qquad \left[\begin{aligned}\cos\alpha &= 1\\ \cos\beta &= 1\end{aligned}\right.$$

$$\mu = \frac{\sin\alpha}{\sin\beta} = \frac{t}{a}\,\frac{\frac{c}{r} + \sin\beta}{\sin\beta} - \frac{c}{r\sin\beta}$$

$$= \frac{tc}{ra\sin\beta} + \frac{t}{a} - \frac{c}{r\sin\beta} \qquad \left[\begin{aligned}\sin\alpha &= \alpha, \text{ etc.}\\ \beta &= \frac{\alpha}{\mu}\end{aligned}\right.$$

$$= \frac{tc\mu}{ra\alpha} + \frac{t}{a} - \frac{c\mu}{r\alpha}$$

$$\mu\left(1 - \frac{tc}{ra\alpha} + \frac{c}{r\alpha}\right) = \frac{t}{a}$$

$$\therefore \quad \mu = \frac{t}{a}\cdot\frac{ra\alpha}{ra\alpha - tc + ta})$$

$$= \frac{t}{a}\cdot\frac{r - a}{r - t}$$

$$\left[\begin{aligned}
\tan(\gamma + \alpha) &= \gamma + \alpha = \frac{d}{u}\\
\tan\gamma &= \gamma = \frac{c}{r}\\
\therefore \quad \alpha &= \frac{d}{u} - \gamma = \frac{d}{u} - \frac{c}{r} = \frac{dr - cu}{ur}\\
\frac{c}{a} &= \frac{d}{u}
\end{aligned}\right.$$

$$\text{If } r = \infty, \quad \mu = \frac{t}{a}$$

Check your determination of μ by using it to calculate F of the lens, and test by observation.

Similarly for a concave surface, we would have [since the angle $(\gamma + \alpha)$ becomes replaced by $(\alpha - \gamma)$ and $\alpha = \frac{d}{u} + \gamma$]

$$\mu = \frac{t}{a} \cdot \frac{r + a}{r + t}$$

PRACTICAL SUGGESTIONS

156. To Focus Accurately. — Set a fine pin (needle) in line with the lens, and with the eye in line fix the vision on the pin. Adjust the lens backward or forward until a motion of the head slightly sidewise does not alter the position of the pin on the image seen through the lens. If on moving the head the pin moves across the image in the same direction, the image (and therefore the lens) is too close, and vice versa.

Better still, use a short focus lens, focussing it on the edge of a translucent (transparent) screen (piece of celluloid). Then move the lens until the image appears distinct, testing similarly by the motion of the head.

157. To Measure an Angle without Angular Instruments. — Determine three points, A, at the vertex of the angle, B, in line with one side of the angle, and C, in line with the other side and at right angles to AB.

This may be done on the ground, or on a table or top of a level box. Then

$$\tan A = \frac{BC}{AB} \quad \text{or} \quad \sin A = \frac{BC}{AC}$$

whence the angle A can be found by a table of tangents or sines.

158. Make the experimental observations with monochromatic light (as well as white) by using red, blue, green screens (colored glass).

Use no alcohol or other solvent on mounted lenses, except in an emergency and with the greatest care. Cleanse greasy lenses with a weak solution of washing soda, rinsing with clean water.

Keep all lenses covered from dust; keep a cover over the eyepiece of the microscope when not in use.

Keep lenses out of the sun as much as possible — they will gradually discolor. The cement may overheat.

Clean lenses with the greatest care, lightest pressure, softest cloth *free from dust* and grit, with a circular motion, never across the lens. Use soft camel's hair brush when feasible.

159. Home-Made Optical Bench. — A little skill can make a home-made optical bench, as shown, with which fairly good work can be done. In the absence of anything better, lenses fastened on the tops of corks with pins will do roughly good work.

a, board on edge ($\frac{1}{2}$ or $\frac{3}{8}$ in. wide), or graduated yard stick.

b, pedestal to keep board upright, two or *more*.

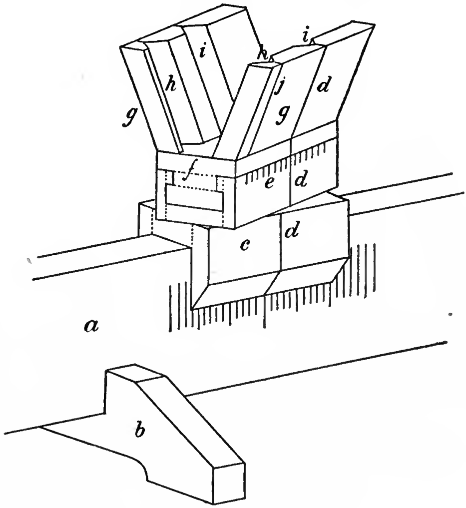
c, sliding piece on top of board, with center line for reading distances moved.

d, center line of the various pieces.

e, rotating piece, pivoted to *c* at center, by small screw or pivot, exactly in center, and in line with center line *d*.

f, lens carrier sliding on *e*, so as to allow bringing the nodals over the center of revolution.

g, *g*, sloping sides to accommodate different sized lenses.



h, *h* / *i*, *i* — grooves in which to slip the lens so that it will be held upright: *h* is used for a short focus lens, when *i* would be too far from the end of *f*, the nearest approach of the screen.

j, center line of *h h*, a known distance from *d*.

Suggestions as to how to put the pieces together are indicated by dotted lines.

Modified carriers (without rotation) should be provided for holding screens, reading lenses (§ 156), etc., to be used in connection with the one above.

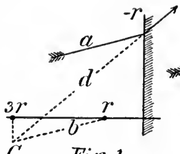


Fig. 1

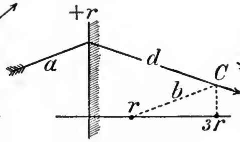


Fig. 2

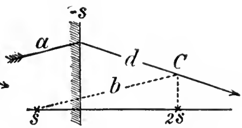


Fig. 3

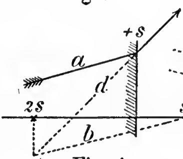


Fig. 4

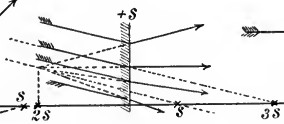


Fig. 5

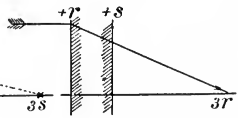


Fig. 6

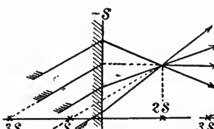


Fig. 7

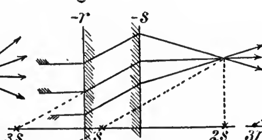


Fig. 8

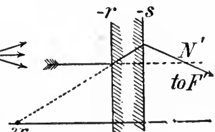


Fig. 9

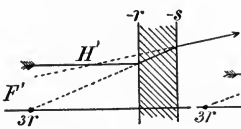


Fig. 10

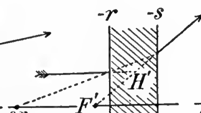


Fig. 11

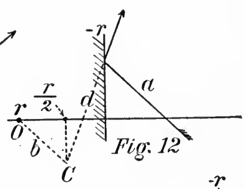


Fig. 12

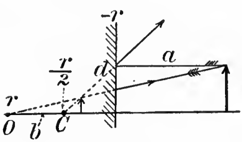


Fig. 13

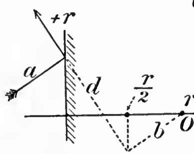


Fig. 14

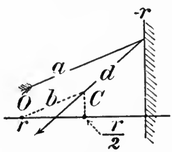


Fig. 15

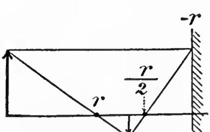


Fig. 16

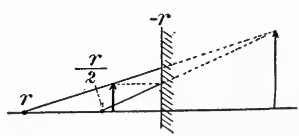


Fig. 17

APPENDIX

This contains a series of progressive propositions giving succinct methods of construction and interpretation, with important theorems for surfaces and lenses, culminating in propositions XIII–XVIII, which give general discriminations for locating the nodals and foci for different forms of lenses.

These propositions afford a valuable general check upon the calculations and graphical constructions, guided as they are by the numerical relations between the surfaces.

The calculator cannot have too many checks, as he will quickly discover when he essays an independent and uncorroborated investigation.

These propositions give a complementary point of view to that in the body of the text, valuable and almost indispensable, *especially for those making original numerical investigations.*

SURFACE REFRACTION

Notation. — *Prolong* is the prolongation on the right of the surface (light is always supposed to come from the left) of the ray impinging on the left side.

Emerge is the position of the ray after refraction through the second surface from the denser to the rarer medium.

r = the radius of the surface of incidence, the surface to the left of the denser medium.

s = the radius of the surface of emergence, the surface to the right of the denser medium.

e = thickness of the lens.

H', F' = nodal of emergence and corresponding focus.

H, F = nodal of incidence and corresponding focus.

[For convenience of illustration, μ is assumed = $\frac{3}{2}$. If μ

is not equal to $\frac{3}{2}$, in the results which follow substitute $\frac{\mu}{\mu - 1}$ for 3, and $\frac{1}{\mu - 1}$ for 2.]

I. To trace an incident ray, surface refraction. Figs. 1 and 2. (Conf. § 33.) Note carefully the order of the letters, which indicate the order of construction; and the formula for construction, $rs \rightarrow \phi_2 \,||$.

II. To trace an emergent ray, surface refraction. Figs. 3 and 4.

III. (Cor. to II.) Fig. 5. Emergent rays from a surface with a + radius, with the prolongs convergent to a point on the axis

beyond the $3\,s$ point, are bent upward, above the horizontal;

at the $3\,s$ point, emerge horizontal;

within the $3\,s$ point

and *beyond* the s point, are bent upward above the prolong, emerging convergent:

and *within* the s point, are bent downward, below the prolong, emerging convergent.

IV. (Cor. to III.) Emerges originating from points on the axis to the left of a + radius surface are divergent, bent upward from the prolong.

V. (Cor. to III.) Fig. 6. If $3\,s > 3\,r - e$ (positive meniscus, convex to the rays), then incident horizontal rays emerge convergent, but

rising from the prolong, if $s < 3\,r - e$;

falling from the prolong, if $s > 3\,r - e$.

If $3\,s < 3\,r - e$ (negative meniscus, convex to the rays), then the incident horizontal rays emerge divergent, rising from the prolong.

VI. (Cor. to II.) Fig. 7. Emerges from a − radius surface, originating from a point on the axis to the *left* of the $3\,s$ point, are bent downward to convergence:

from the $3\,s$ point, are bent downward to horizontality;

from a point *within* the $3\,s$ point, are divergent, falling below the prolong, if from *without* the s point; rising above the prolong, if from *within* the s point.

VII. (Cor. to VI.) Rays convergent to a point on the

right of a − radius surface have their emerges bent down below the prolongs.

VIII. (Cor. to VI and I.) Fig. 8. If $3s - e > 3r$ (negative meniscus, concave to rays), then incident horizontal rays are divergent;

falling from the prolong, if $s < 3r + e$;

rising from the prolong, if $s > 3r + e$.

If $3s - e < 3r$ (positive meniscus, concave to the rays), then incident horizontal rays are convergent, falling below the prolong.

IX. (Cor. to VIII.) Fig. 9. In a positive meniscus, concave to the rays, ($s < r + e/3$), the nodal of emergence is outside the lens to the right, and F′ to the right of that.

X. (Cor. to VIII.) Figs. 10, 11. In a negative meniscus, concave to the rays ($s > r + e/3$), the nodal of emergence is on the outside of the lens to the left if $s < 3r + e$; or in the lens if $s > 3r + e$. In either case F' is to the left of H'.

XI. (Cor. to V.) In a positive meniscus, convex to the rays, since $3s > 3r - e$, the nodal of emergence is outside the lens to the left if $s < 3r - e$; inside the lens if $s > 3r - e$.

In either case the focus F' is to the right.

XII. (Cor. to V.) In a negative meniscus ($3s < 3r - e$), convex to the rays, since the emerge rises from the prolong, the nodal of emergence is to the right and outside, and F' to the left.

XIII. (Cor. to I and IV.) Double concave lens. Emerges resulting from incident horizontal rays are bent upward from the prolong and H' is within the lens, F' to the left. Similarly as to H, F, *mutatis mutandis.*

XIV. (Cor. to I and VII.) Double convex lens. Incident horizontal rays are bent down by the first surface and the emerge falls below the prolong, therefore H' is within the lens, and F' to the right. Similarly as to H, F.

XV. (Cor. to IX and XI.) Positive meniscus, concave to the rays. H' is outside the convex with F' further outside, measured with the rays.

H is outside the convex if $r < 3s - e$.

H is inside if $r > 3s - e$.

F is measured from H toward the concave, against the rays.

XVI. (Cor. to X and XII.) Negative meniscus, concave to rays.

H' is outside the concave if $s < 3r + e$.

H' is inside if $s > 3r + e$.

F' is measured from H' against the ray.

H is outside the concave with F measured with the rays.

XVII. (Cor. to X and IX.) Positive meniscus, concave toward the rays.

H' is outside the convex if $s < 3r - e$.

H' is inside if $s > 3r - e$.

F' is measured with the rays.

H is outside the convex with F measured against the rays.

XVIII. (Cor. to XII and X.) Negative meniscus, convex to rays.

H' is outside the concave, with F' measured against the rays.

H is outside the concave if $r < 3s + e$.

H is inside if $r > 3s + e$.

F is measured with the rays.

SURFACE REFLECTION

(The order of the letters in the diagrams indicates the order of operations. Conf. § 7.)

Reflection from convex surface. Figs. 12, 13, 14. Image virtual and smaller. (Conf. notation of prop. I.)

Reflection from concave surface. Figs. 15, 16. Image real.

Reflection from concave surface. Fig. 16. Object outside the center. Image real, inverted, smaller, inside the object.

Reflection from concave surface. Fig. 17. Object inside the focus ($r/2$). Image virtual, erect, larger, behind the mirror. (For object between the center and focus, image is real, inverted, larger, outside the object.)

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